CEE598 - Visual Sensing for Civil Infrastructure Eng. & Mgmt.

Session 3 – Camera Model and Projective Geometry

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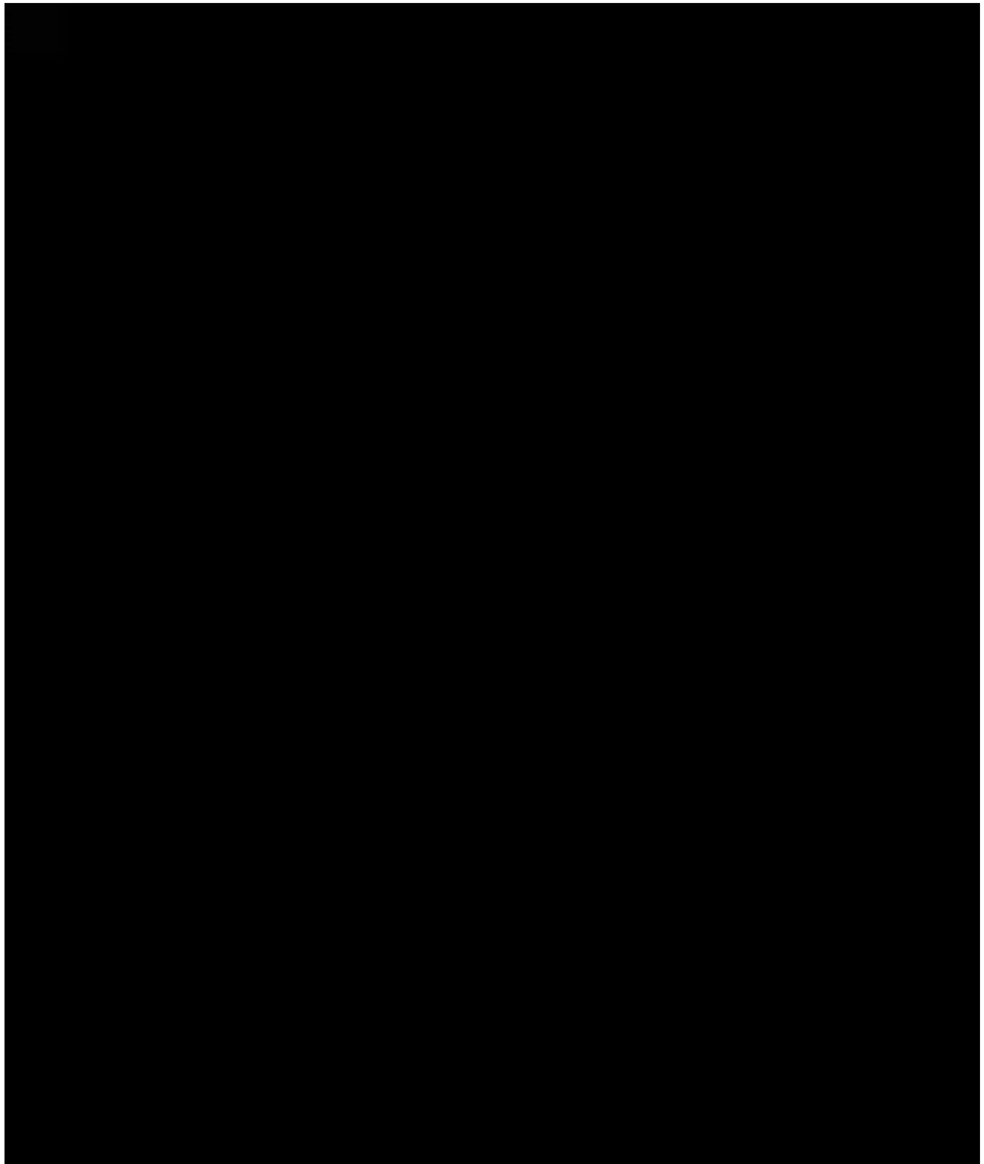
CVPR Best Demo 2009

Stereo-paired iPhone 4 @ 30fps



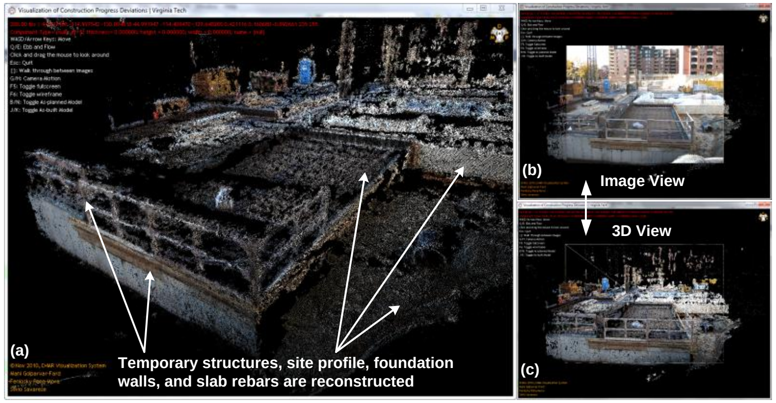
Marilia Maschion	Producer & Director	UCSD ICAM undergraduate
Vincent Rabaud	Technical Advisor	UCSD CSE student
Serge Belongie	Project Advisor	UCSD CSE Department

A00: watch

Illinois Compass2g Site> Course Contents> Supplementary Documents> 3DVision.avi

A0: review SVD decomposition

Outline



Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

- Next Class

Readings:

- [FP] Chapters 1 – 3
- [HZ] Chapter 6

Some of the slides in this lecture are courtesy to Prof. Jon Ponce, University of Illinois, Prof. Silvio Savarese, University of Michigan and Profs. Marc Levoy and Fei-Fei Li, Stanford University.

Outline

Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Without cameras, we wouldn't have Visual Sensing Class ☹

Daily Construction Photos

At the front left-side of the site lies spare sections of the work platforms, presumably to be used around the back of the building when time comes.

Daily Construction Photos

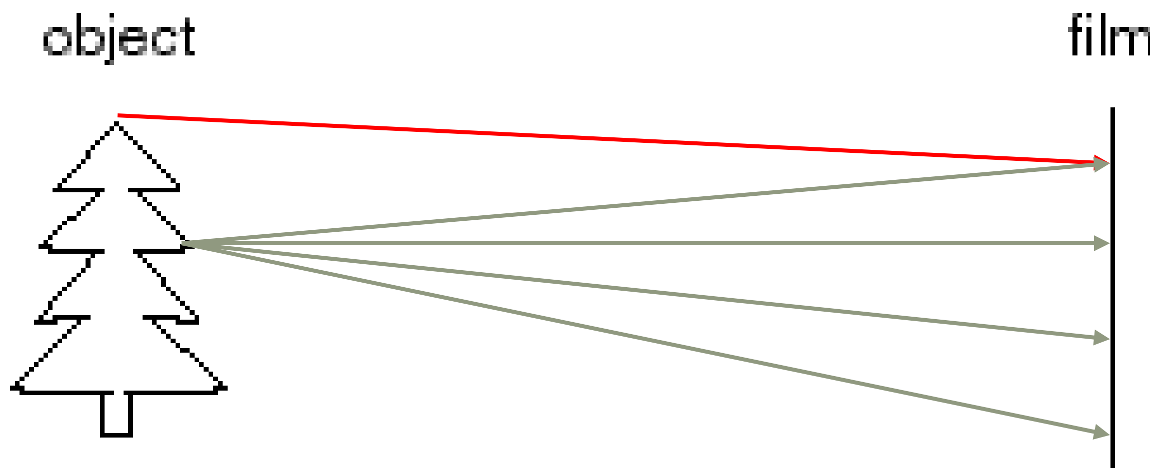

©2011, Construction Performance Improvement Class, Mani Golparvar-Fard (Photo Courtesy of Anthony Lee)

At the first floor, it is clearly not as far along as the top two floors. Here, drywall and insulation installation has not even begun.

Daily Construction Photos

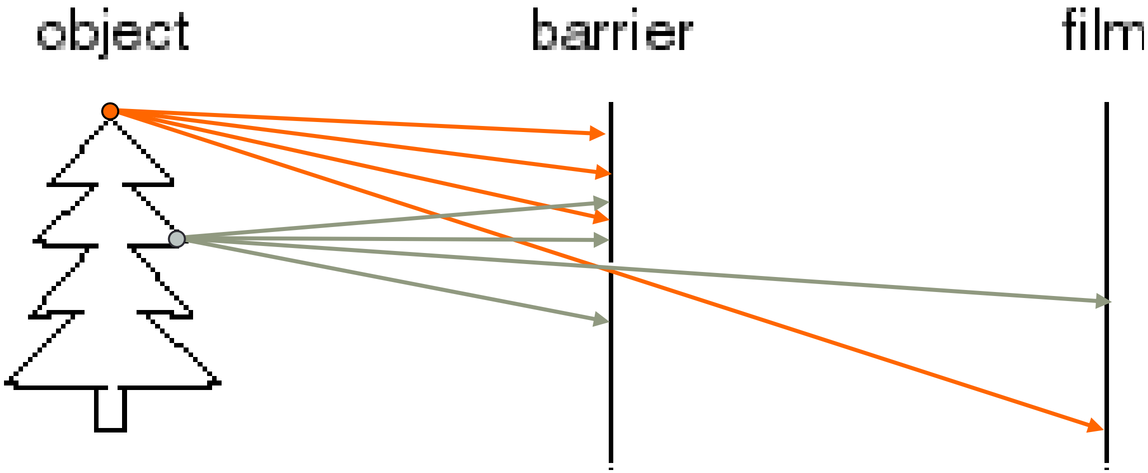
A DEWALT rotary laser is used to pinpoint exact height levels for construction. Also in this picture, one can see the fireproofing material applied on all steel elements of the structure.

How do we see the world?



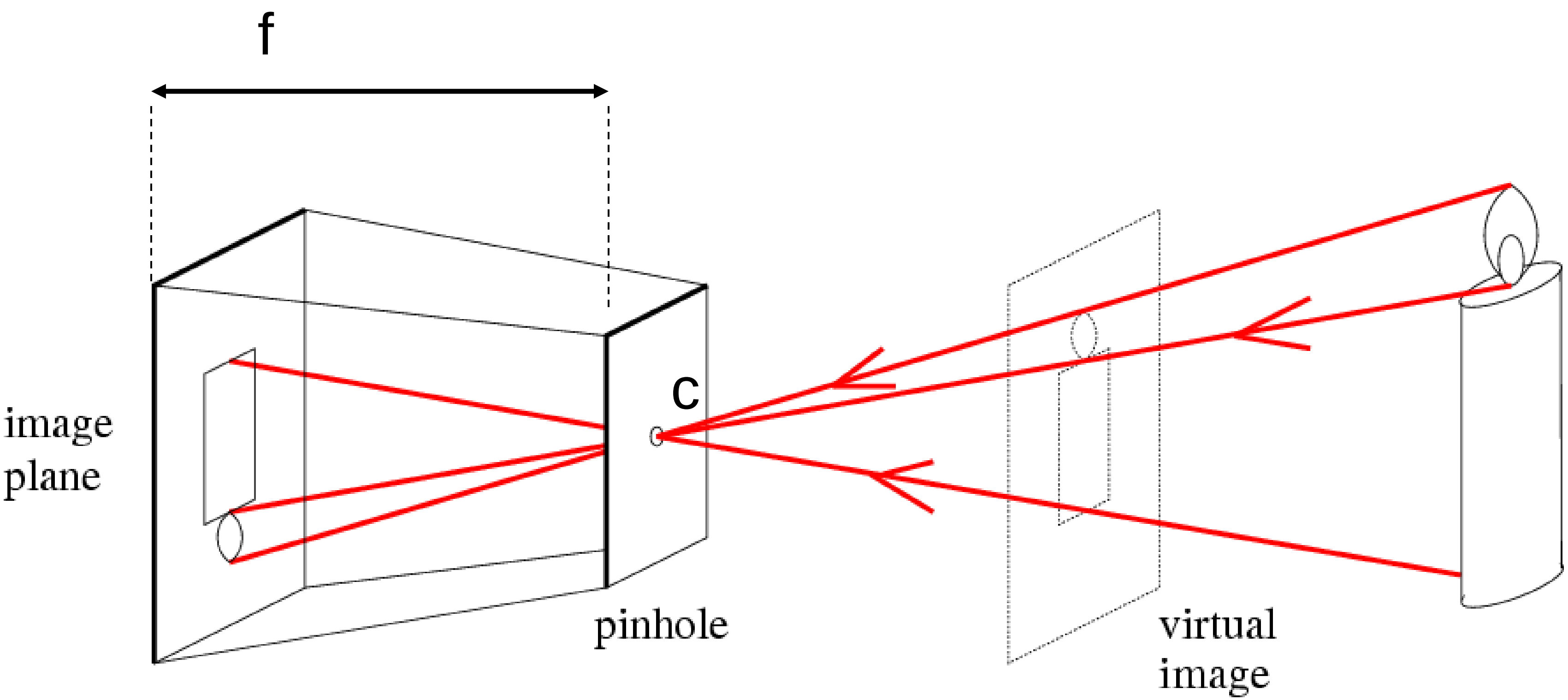
- Let’s design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Pinhole camera

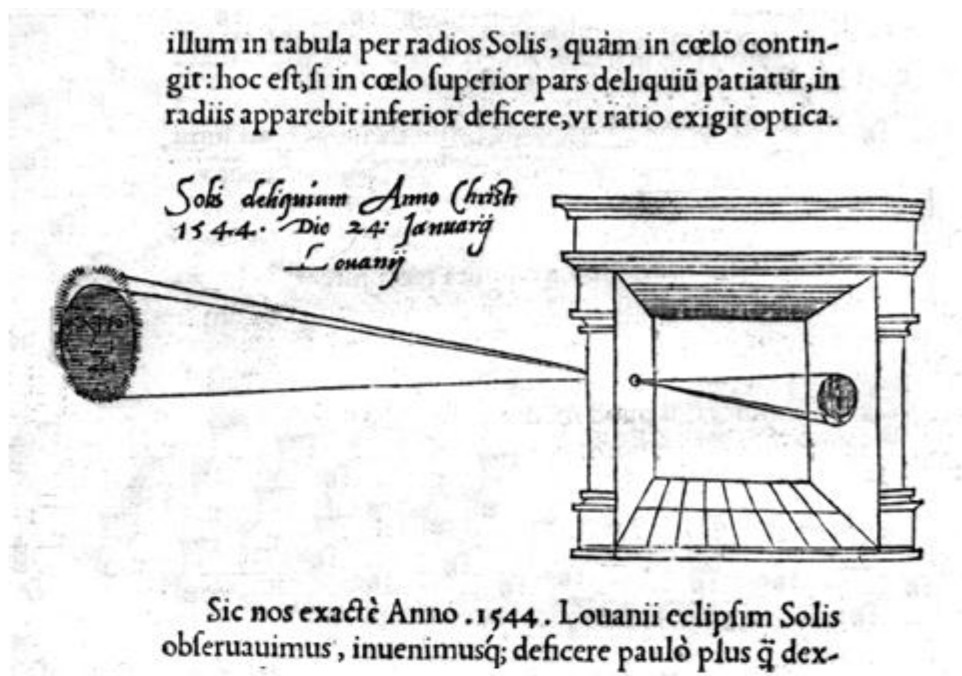


f = focal length
c = center of the camera

Some history…

Milestones:

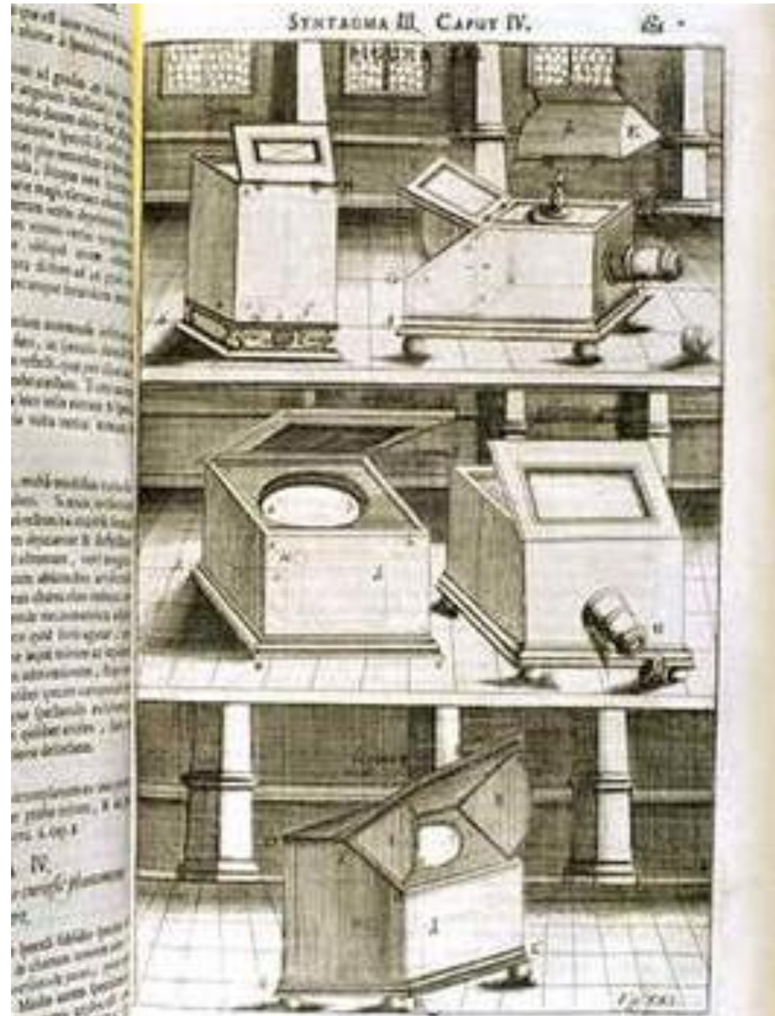
- Leonardo da Vinci (1452-1519): first record of camera *obscura*



Some history…

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera



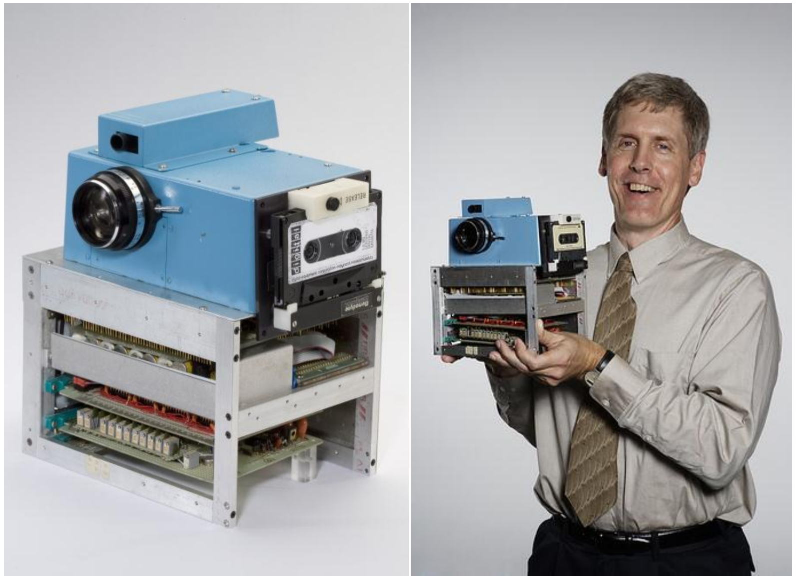
Some history…

Photography (Niepce, "La Table Servie," 1822)

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera
- Joseph Nicephore Niepce (1822): first photo - birth of photography

- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)
- First Digital Camera (Kodak, 1975)
    - Picture of Steve Sasson

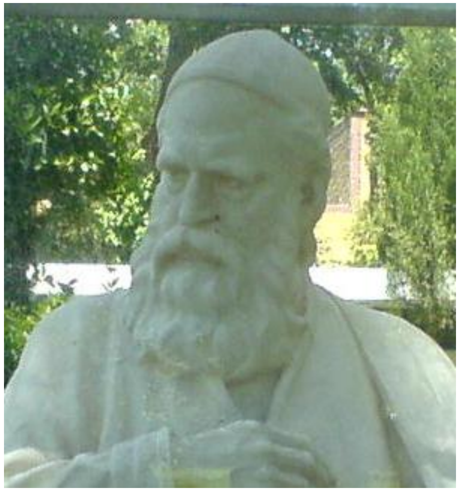


Let's also not forget…

Motzu
(468-376 BC)

Oldest existent book on geometry in China

Aristotle
(384-322 BC)
Also: Plato, Euclid

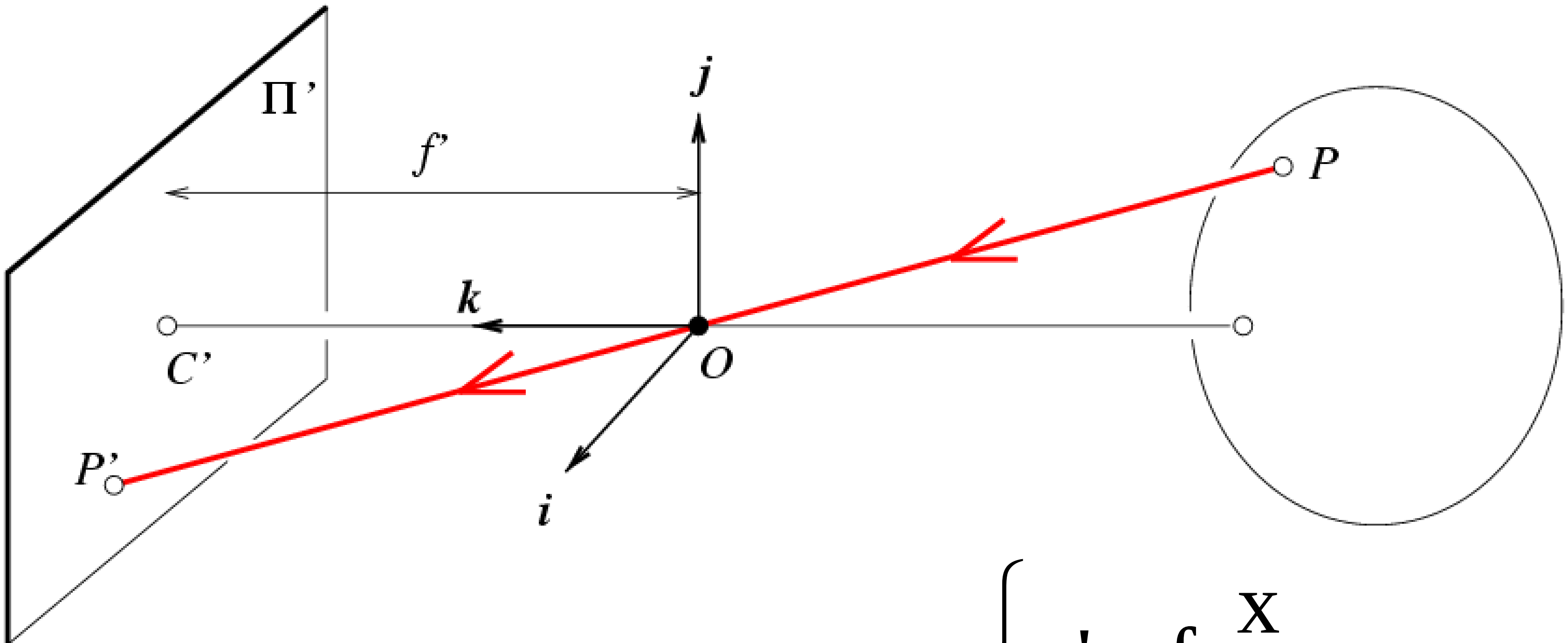


Omar Khayyam
(1080-1131)

Non-Euclidean Geometry
Parallel postulate

Al-Kindi (c. 801–873)
Ibn al-Haytham (965-1040)

Pinhole camera

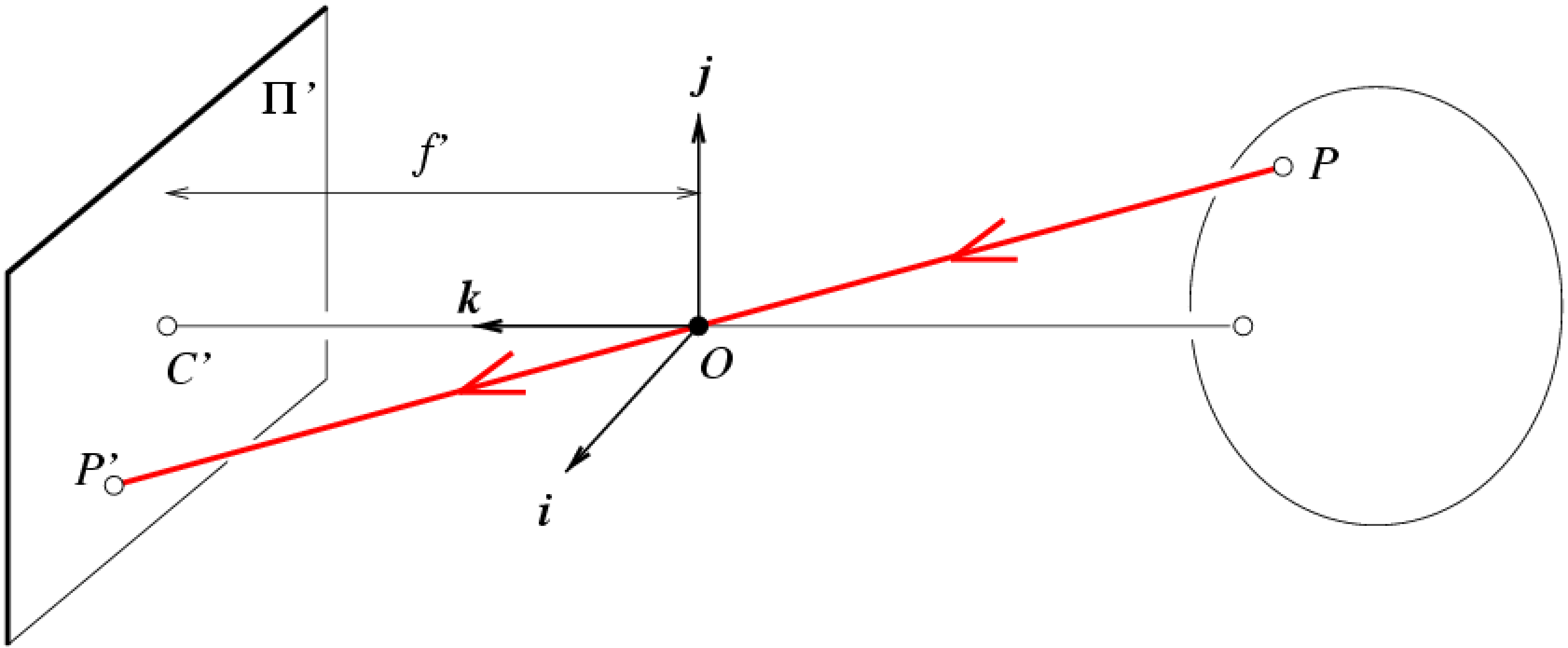


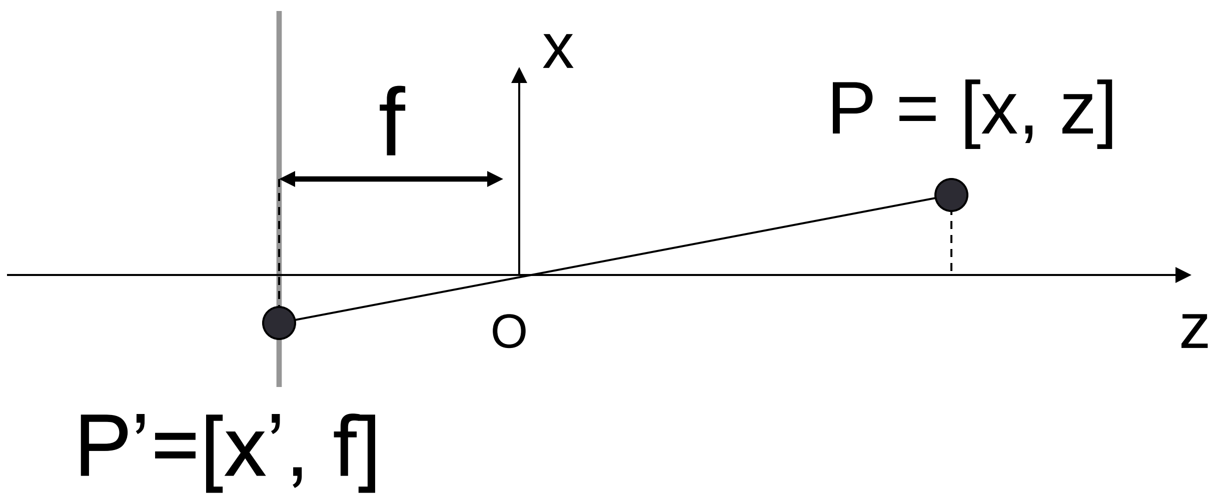
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f \dfrac{x}{z} \\ y' = f \dfrac{y}{z} \end{cases}$$

Derived using similar triangles

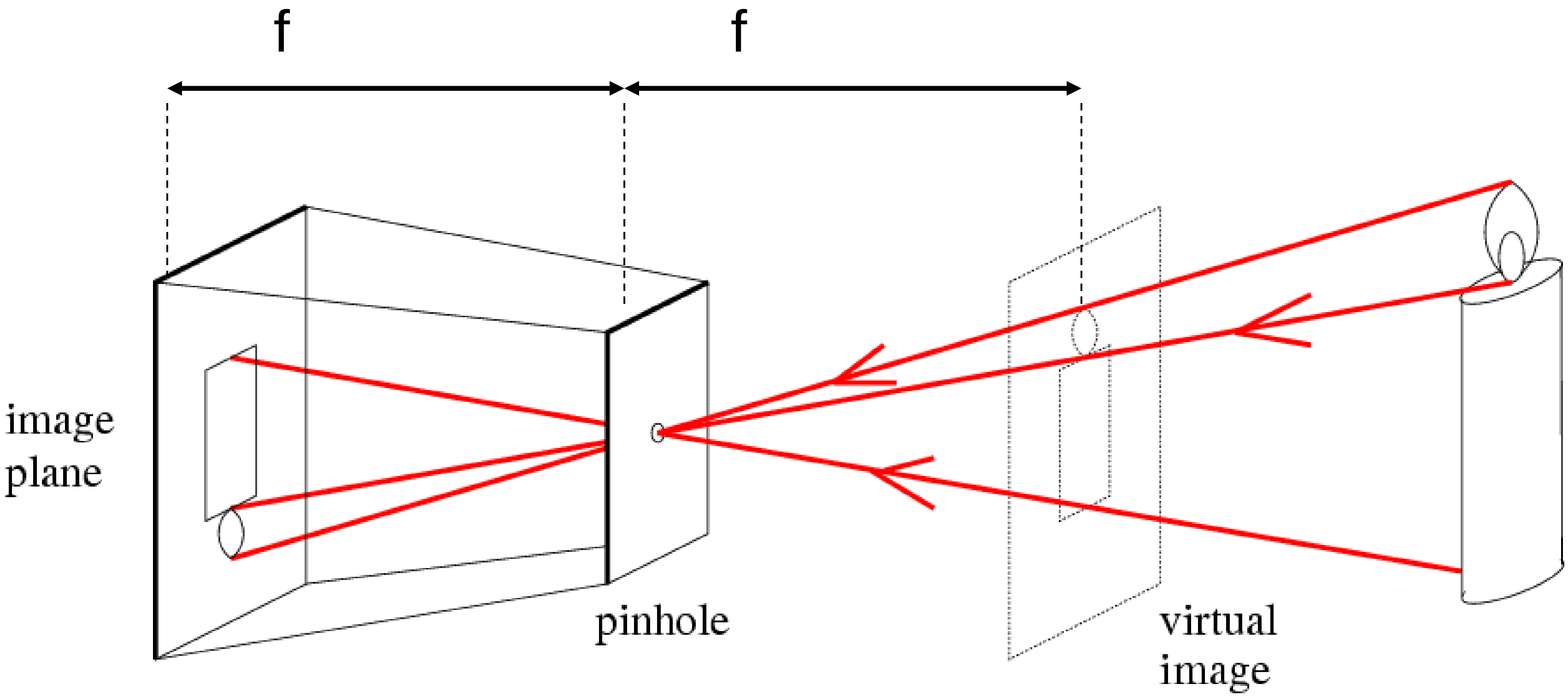
Pinhole camera





$$\frac{x'}{f} = \frac{x}{z}$$

Pinhole camera

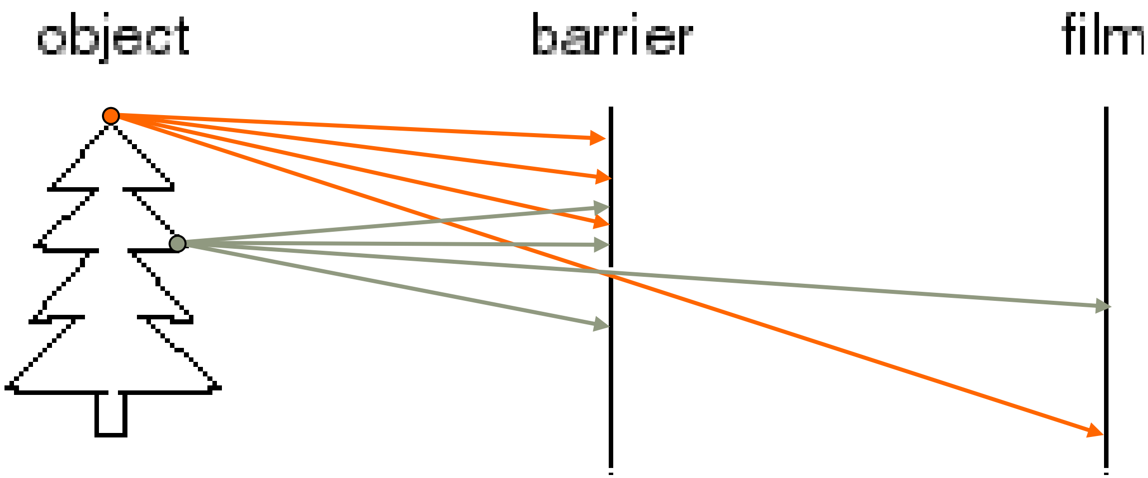


Common to draw image plane *in front* of the focal point. Moving the image plane merely scales the image.

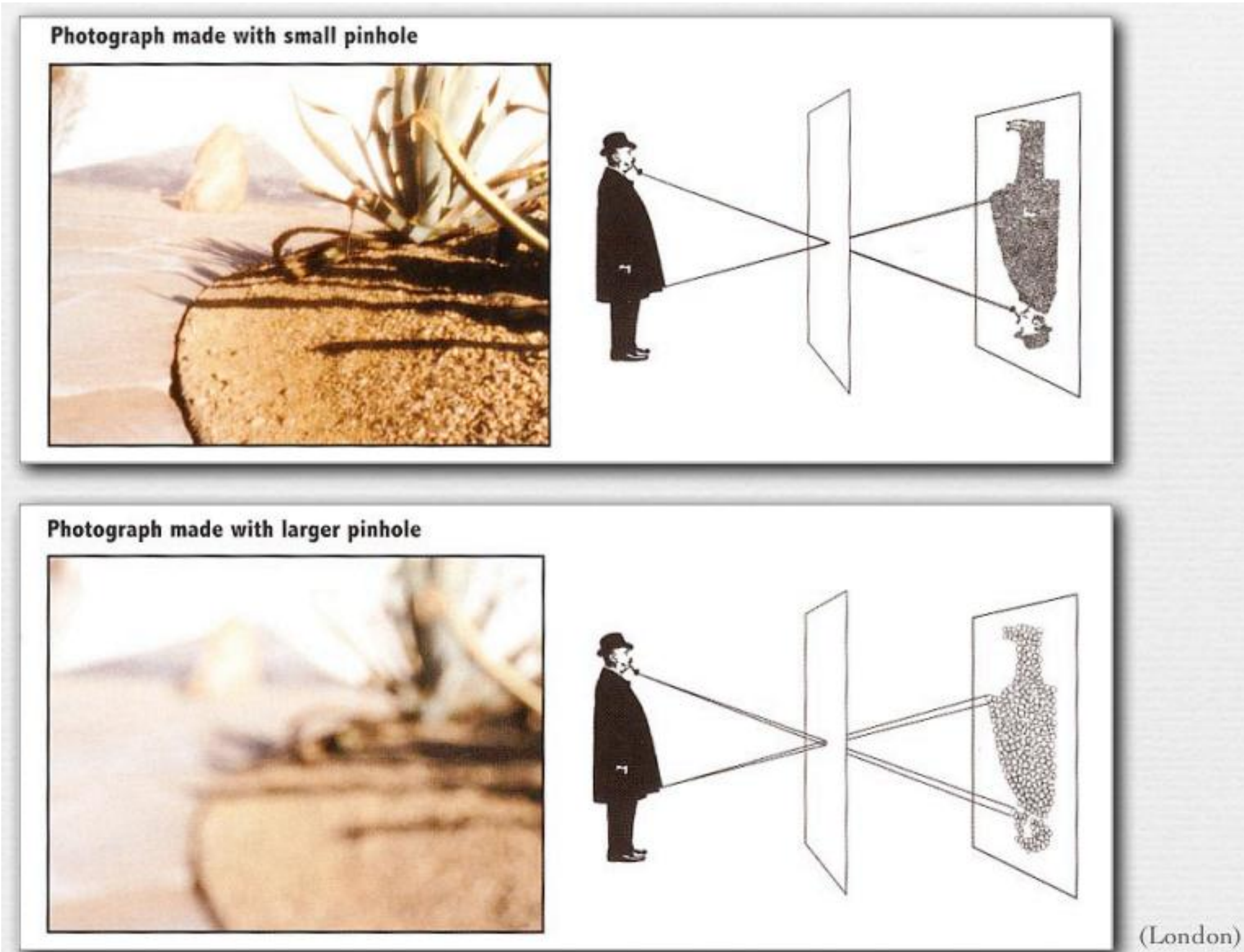
$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Pinhole camera

Is the size of the aperture important?

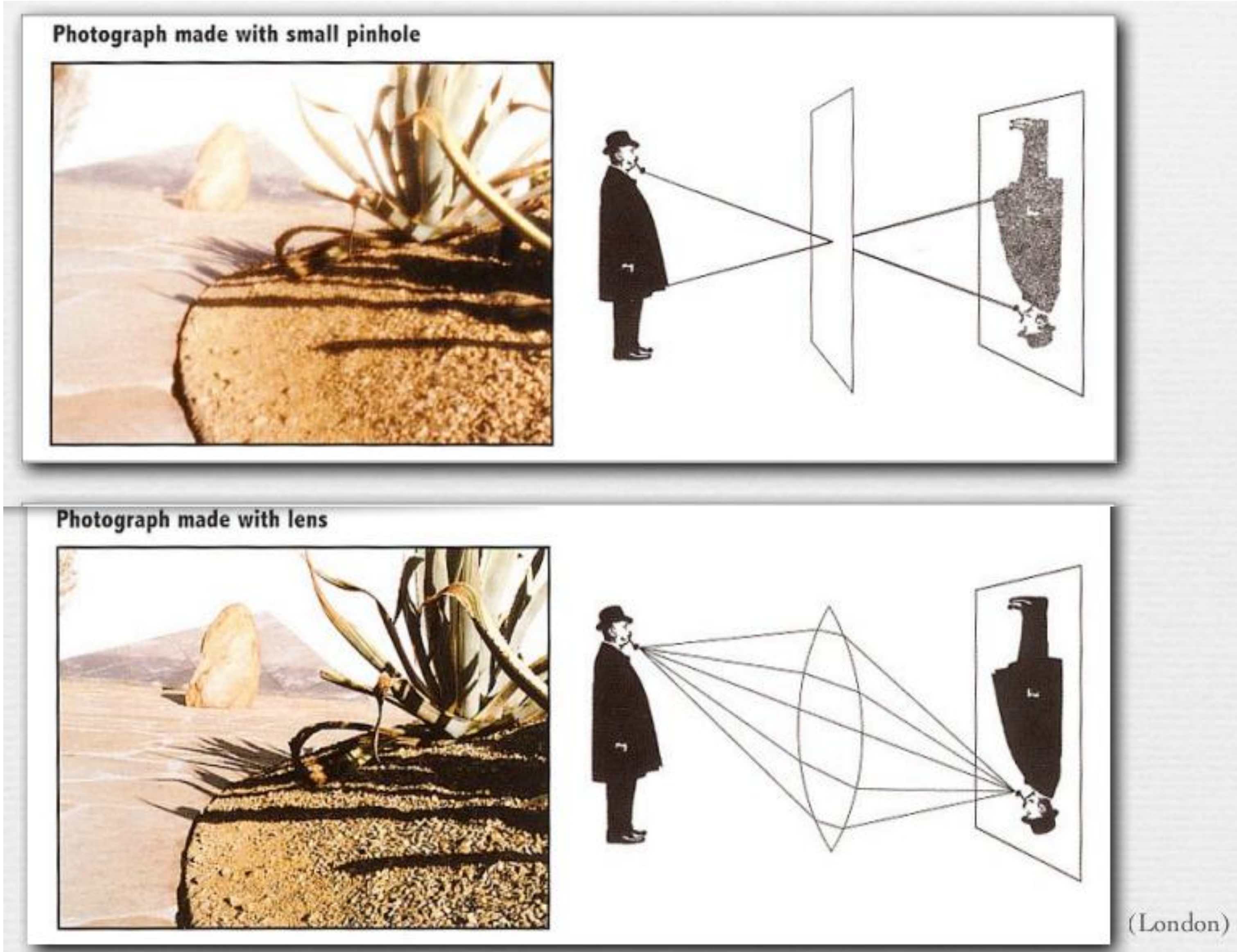


Effect of the Pinhole Size



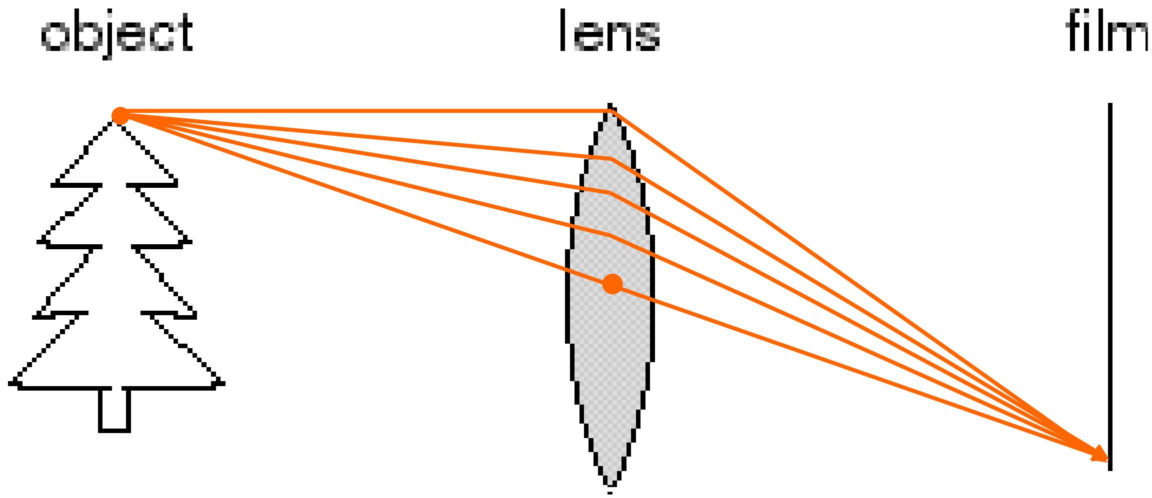
Adapted from Levoy (2011)

Replacing the Pinhole with a Lens



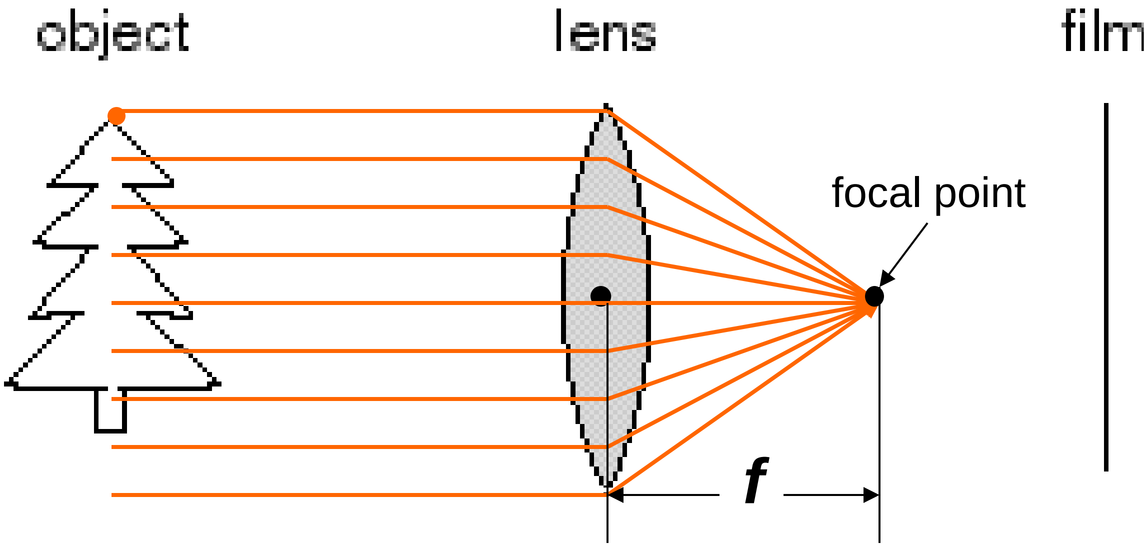
Adapted from Levoy (2011)

Cameras & Lenses



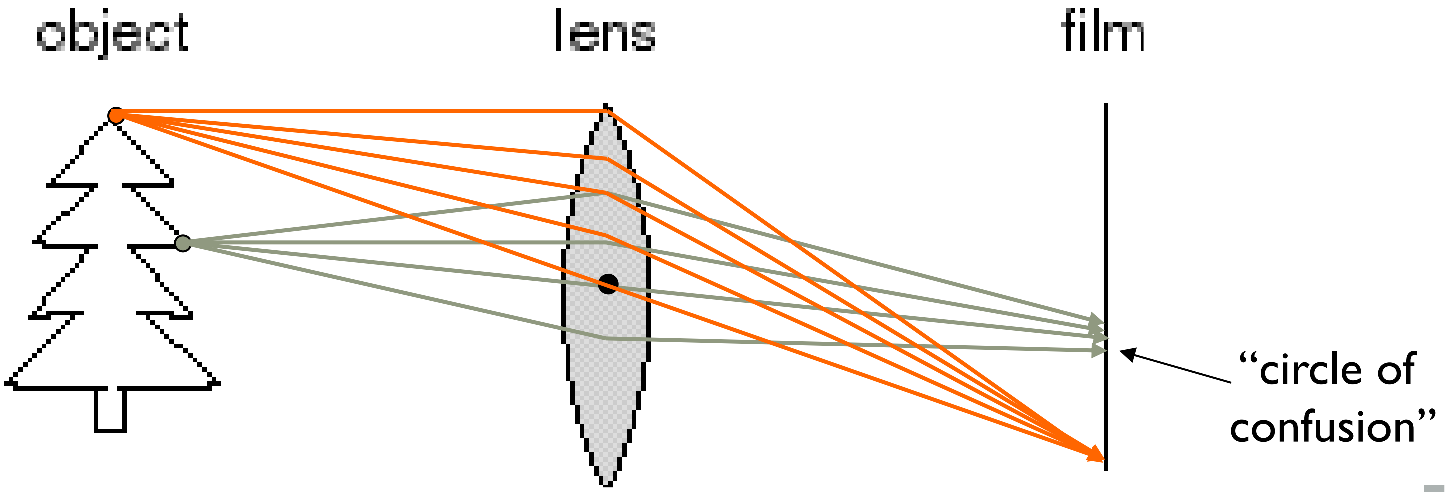
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Cameras & Lenses

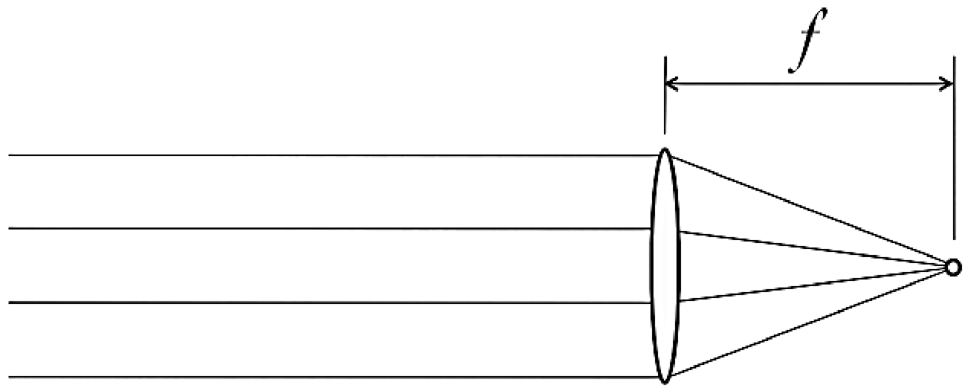


- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"

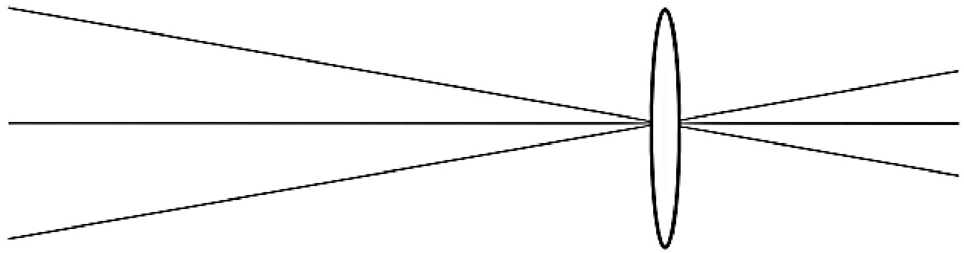
    [other points project to a "circle of confusion" in the image]

Geometrical Optics

- parallel rays converge to a point located at focal length f from lens

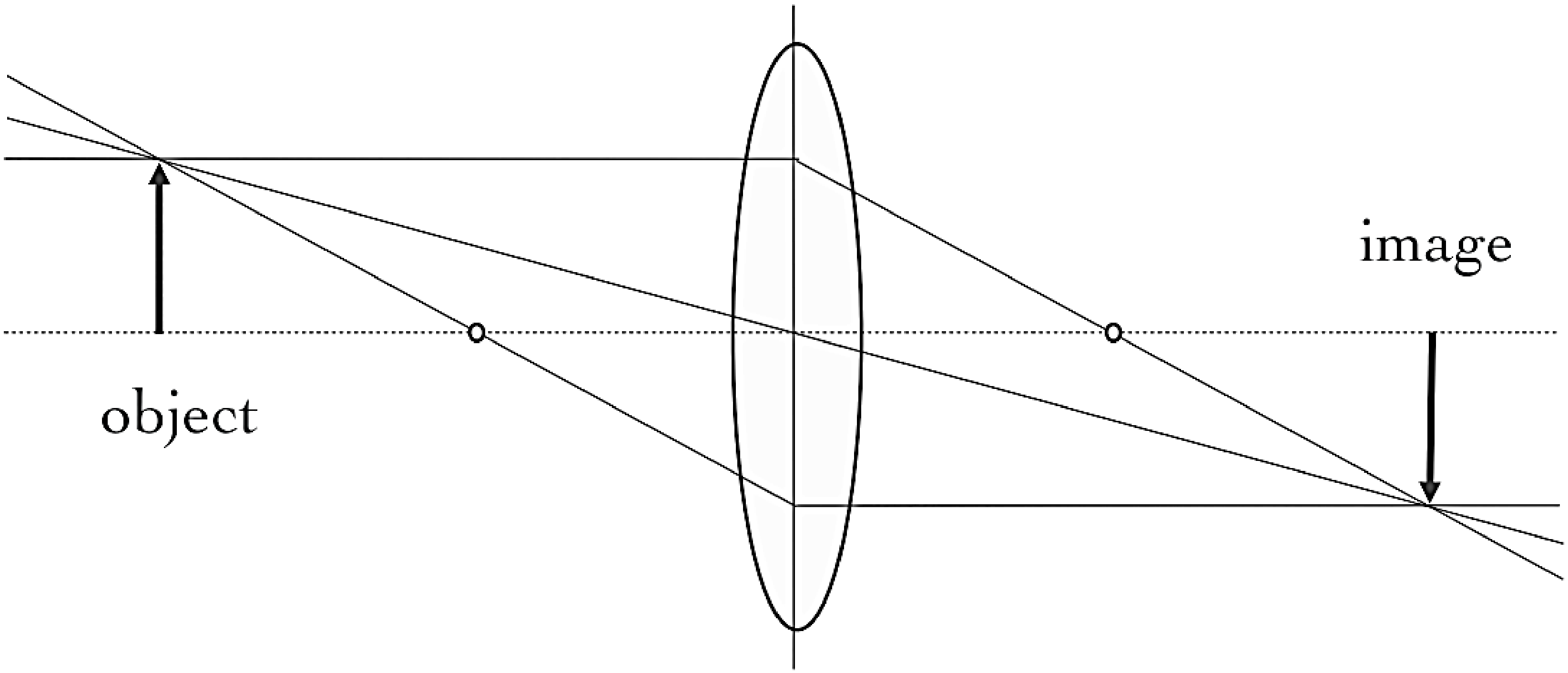


- rays going through center of lens are not deviated
  - hence same perspective as pinhole



Adapted from Levoy (2011)

Gauss' Ray Tracing Construction

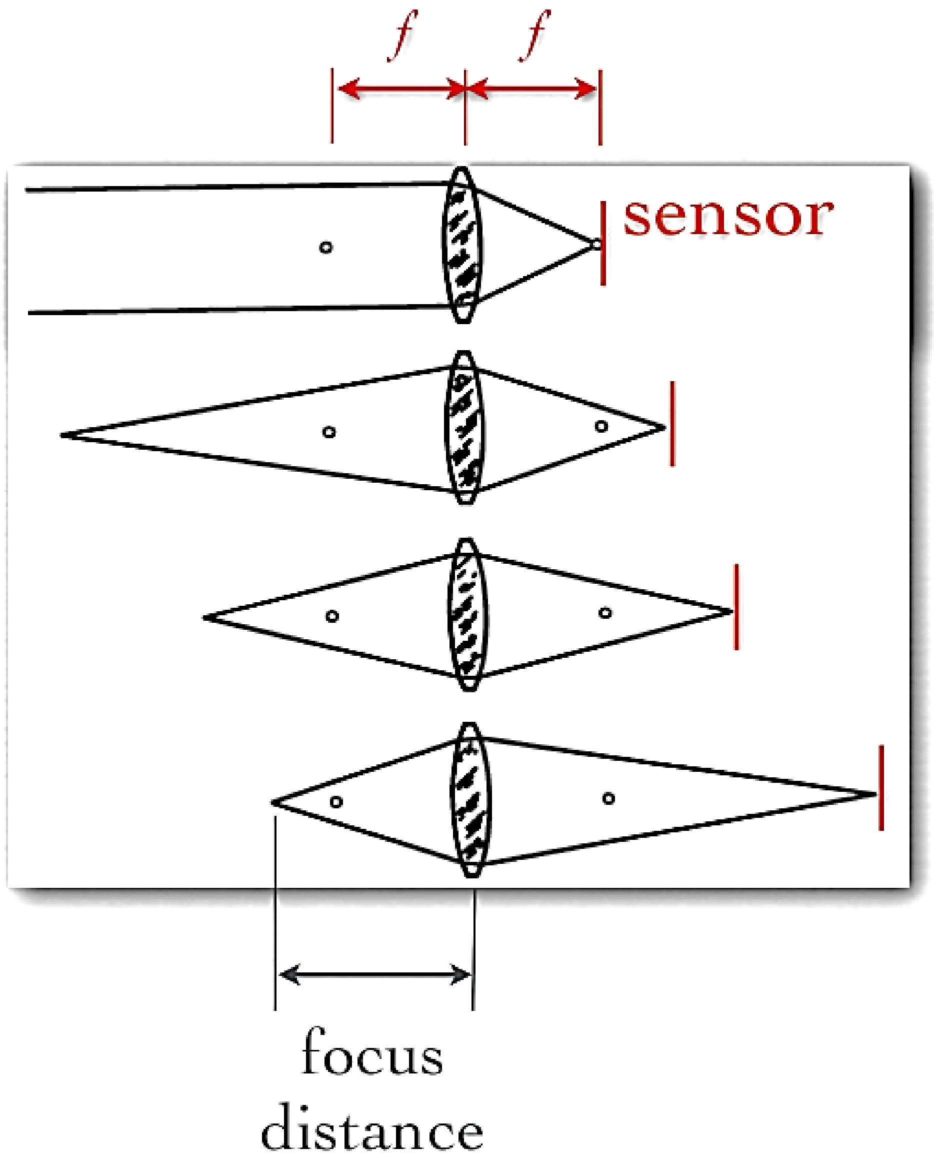


- rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens

Adapted from Levoy (2011)

Changing the Focus Distance

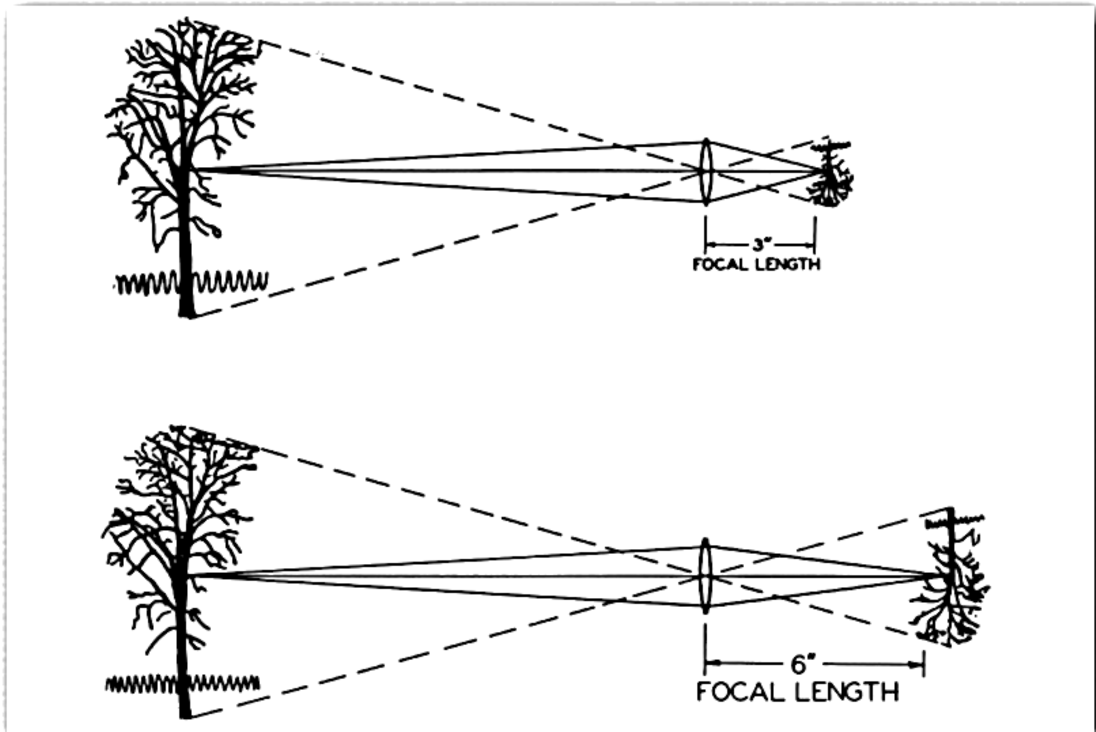
- To focus on objects at different distances, move sensor relative to lens



Adapted from Levoy (2011)

Changing the Focal Length

- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back
- focused image of tree is located slightly beyond the focal length

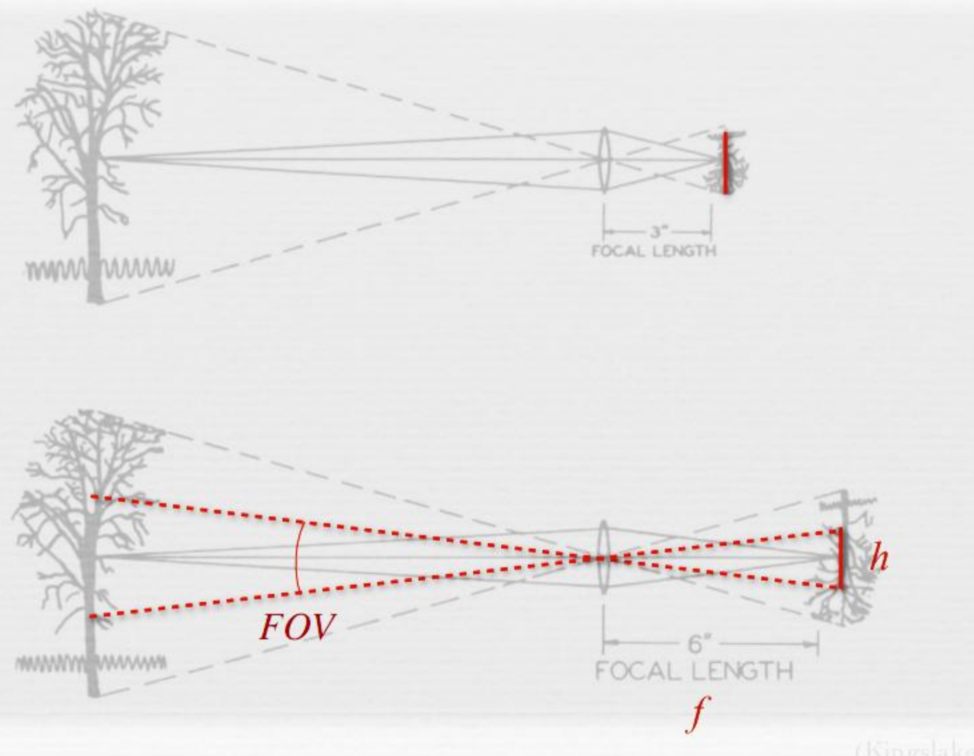


(Kingslake)

the tree would be in focus at the lens focal length only if it were infinitely far away

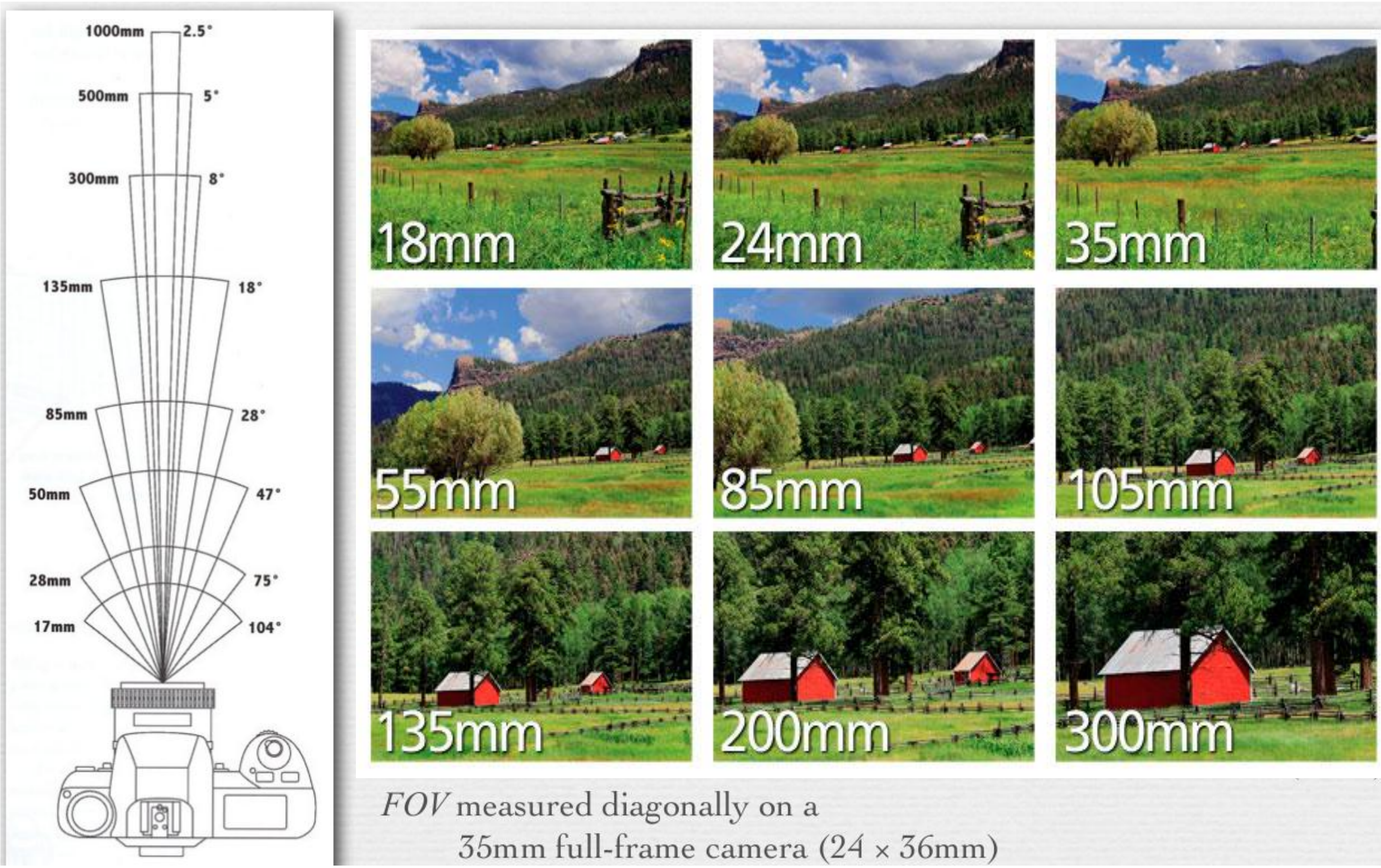
Changing the Focal Length

- if the sensor size is constant, the field of view becomes smaller

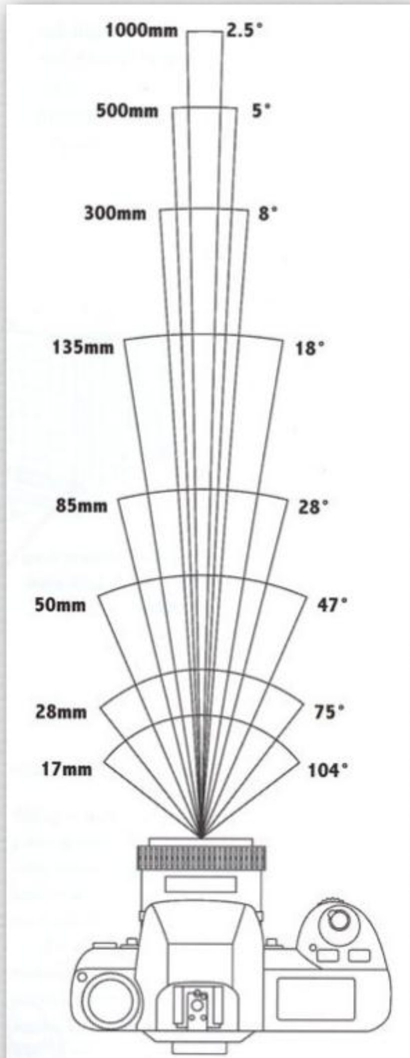


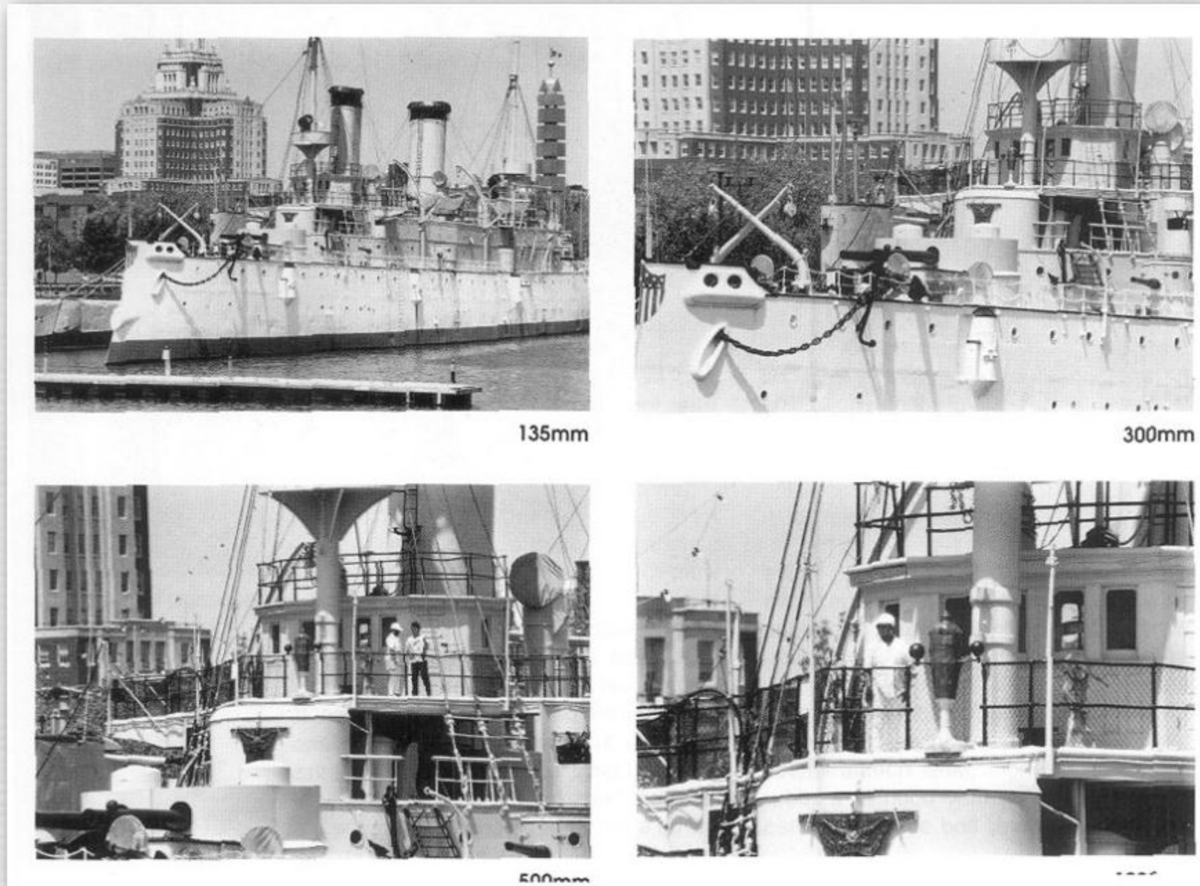
$$FOV = 2\ arctan\ (h\ /\ 2f)$$

Focal Length and Field of View



Focal Length and Field of View





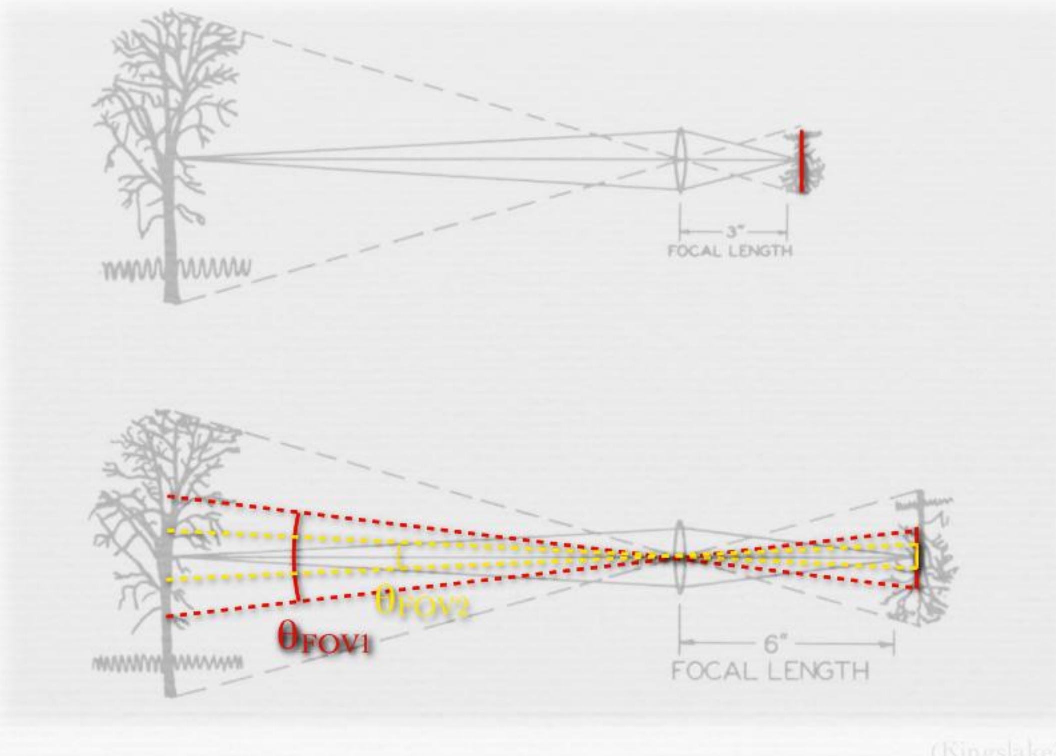
(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

Adapted from Levoy (2011)

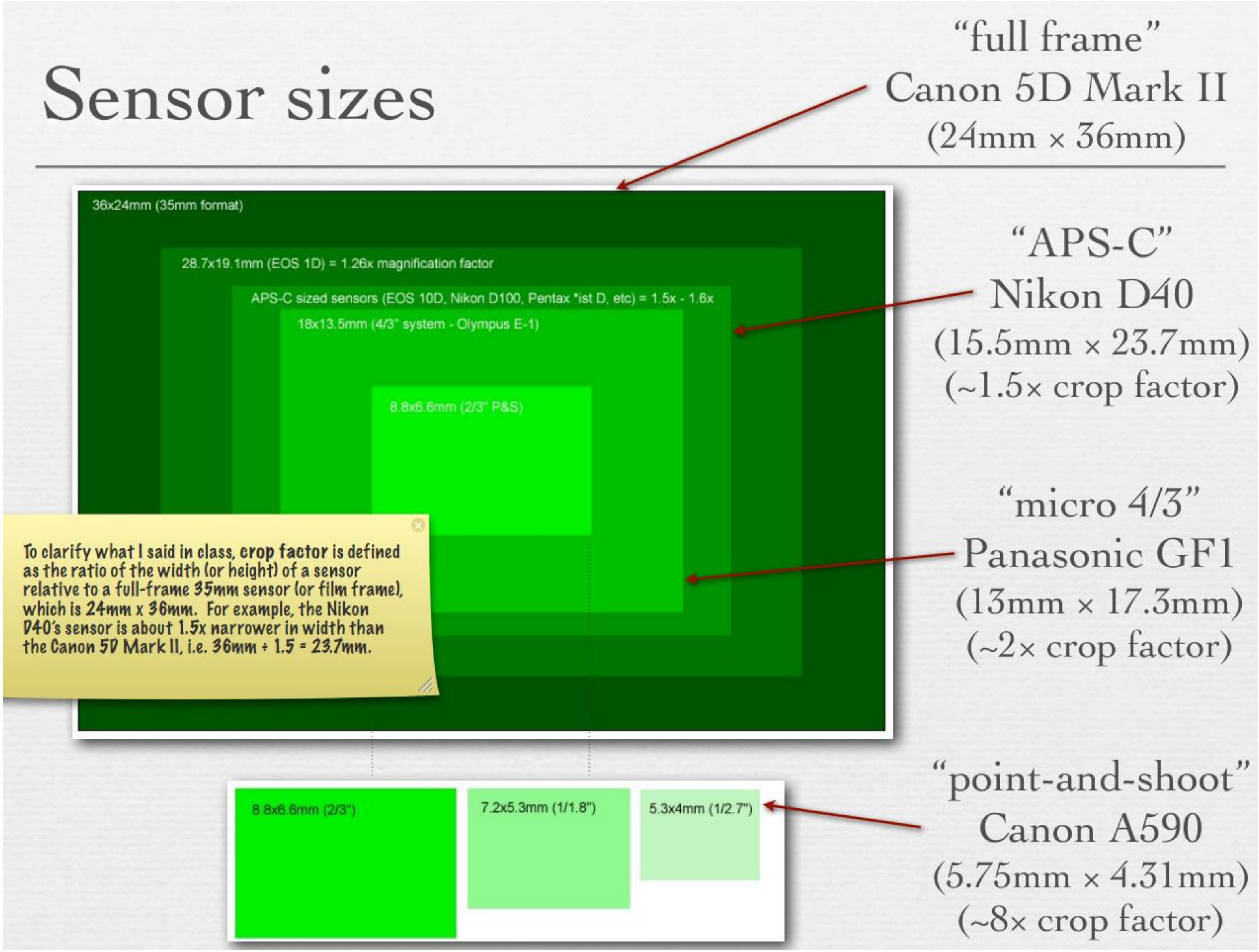
Changing the Sensor Size

- if the sensor size is smaller, the field of view is smaller too
- smaller sensors either have fewer pixels, or noiser pixels



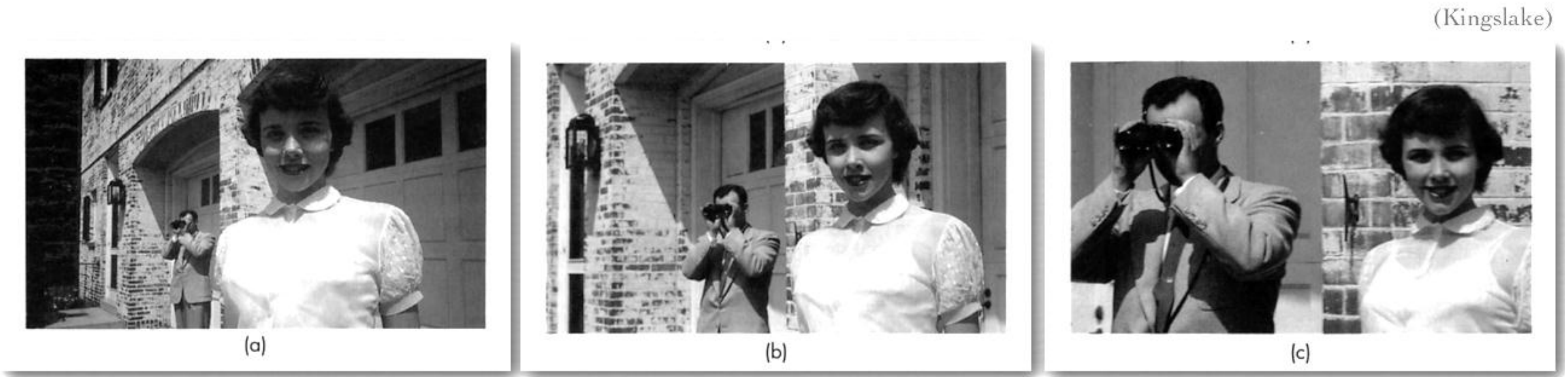
Adapted from Levoy (2011)

Camera Sensor Sizes



Adapted from Levoy (2011)

Changing the Focal Length vs. the Viewpoint



Wide-angle

Telephoto and Moved back

- Changing the focal length lets us move back from a subject, while maintaining its size on the image
- Notice that moving back changes perspective relationships

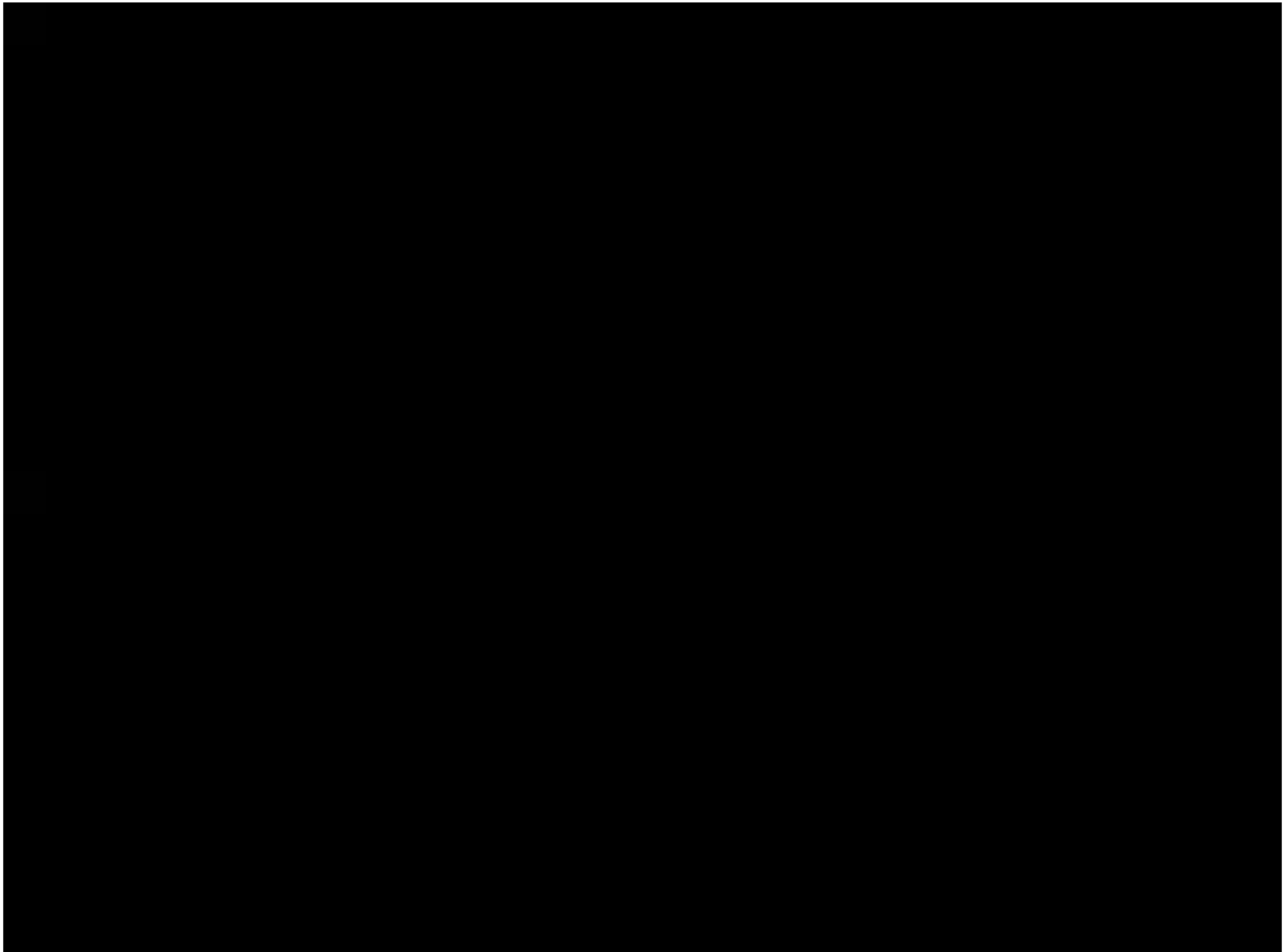
Adapted from Levoy (2011)

Changing the Focal Length vs. the Viewpoint

- Moving back while changing the focal length lets you keep objects at one depth the same size
- In cinematography, this is called the dolly zoom, or "Vertigo effect", after Alfred Hitchcock's movie

Adapted from Levoy (2011)

Changing the Focal Length vs. the Viewpoint



Effect of focal length on portraits

- Standard "portrait lens" is 85mm


Wide angle


Standard


Telephoto

Focal length is getting longer

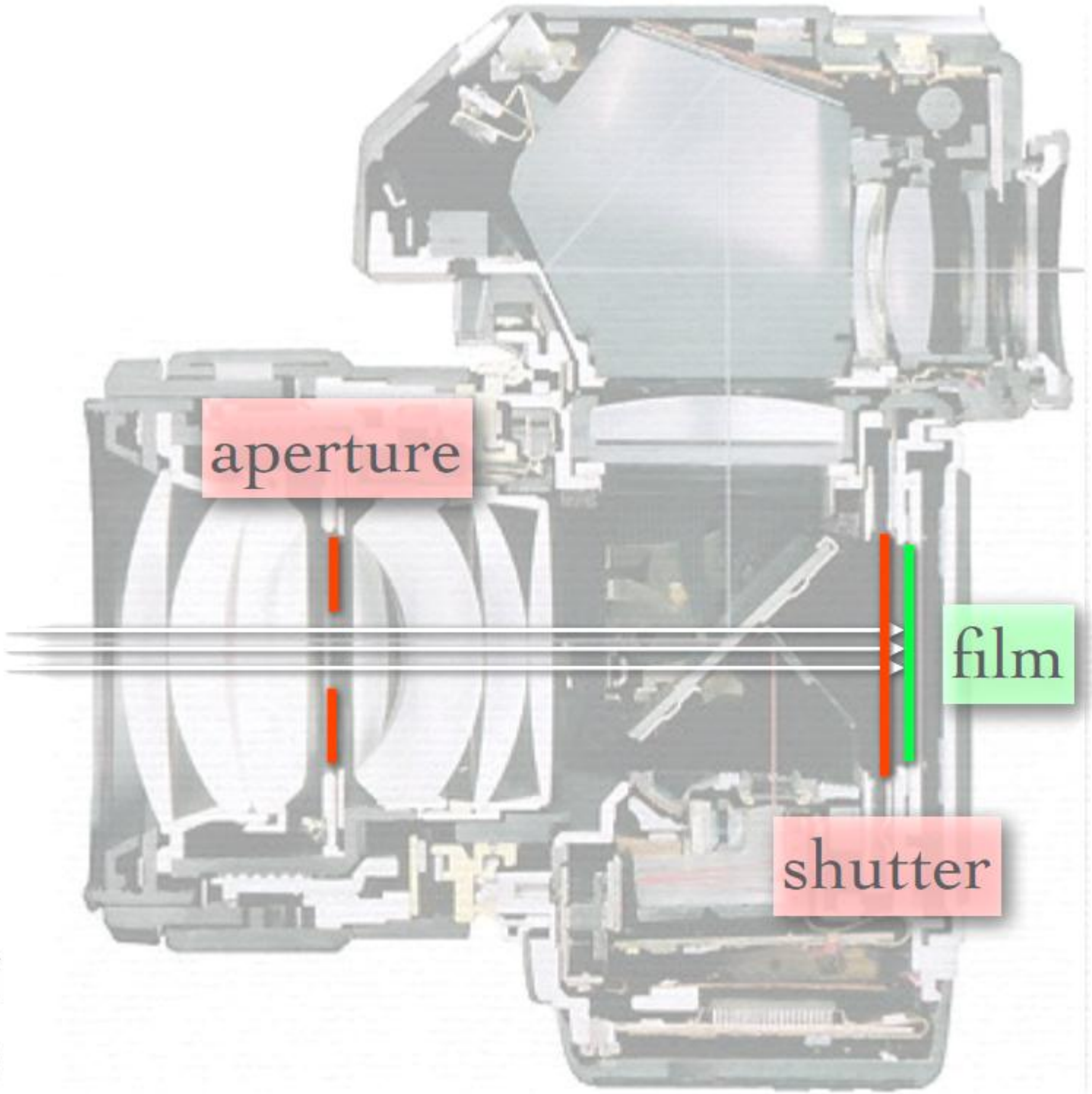
Exposure

- $H = E \times T$
- Exposure = Irradiance × Time

- Irradiance (E)
  - Amount of light falling on a unit area of sensor per second
  - Controlled by aperture

- Exposure Time (T)
  - In seconds
  - Controlled by shutter

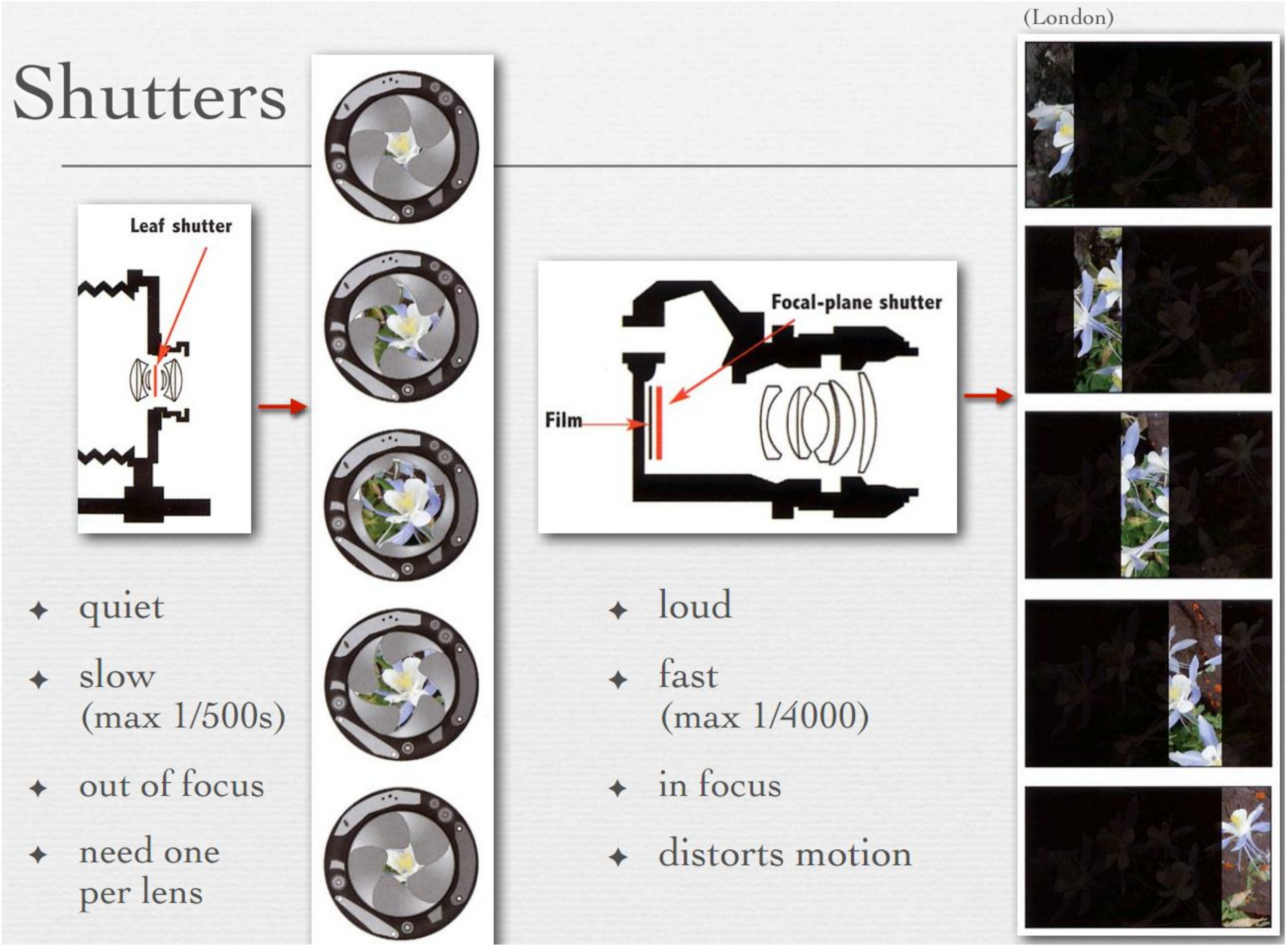
Single Lens Reflex Camera



Nikon F4
(film camera)

Adapted from Levoy (2011)

Shutters



Adapted from Levoy (2011)

Low Shutter Speed

http://www.annedarlingphotography.com/images/slow-shutter-speed-2-0.jpg

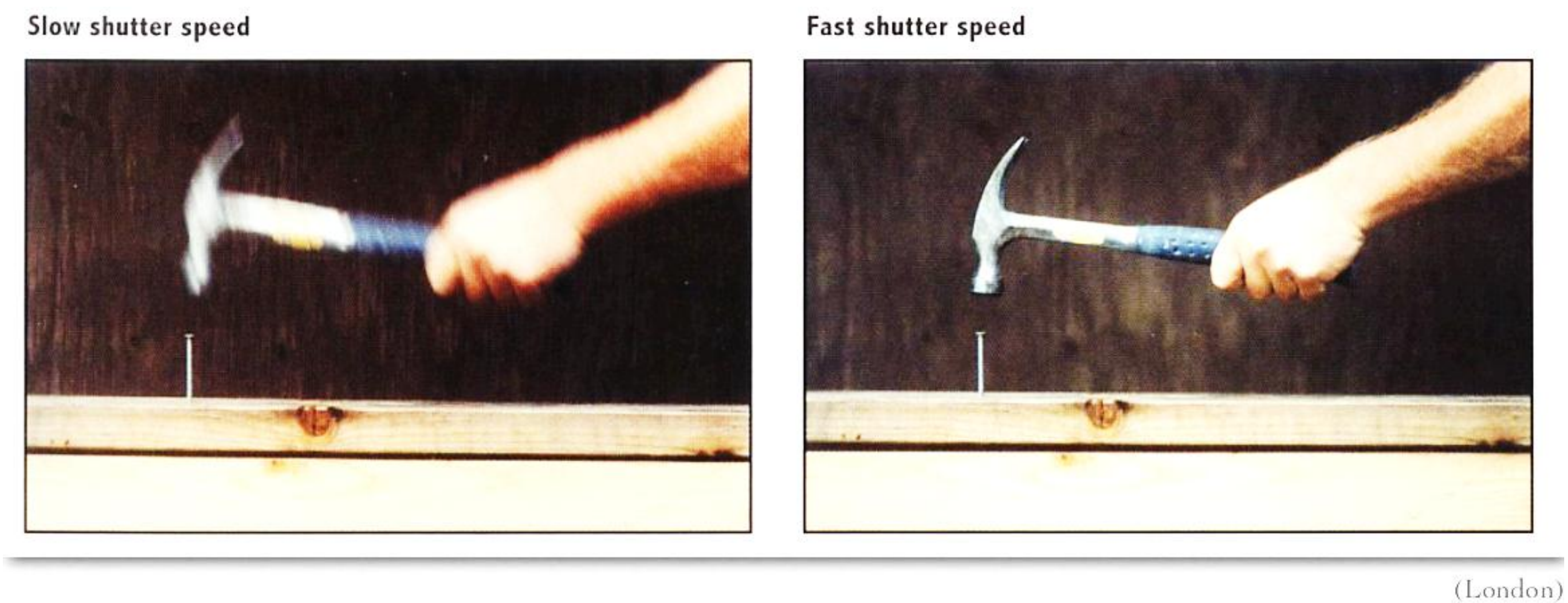
Shutter Speed

- Controls how long the sensor is exposed to light
- Linear effect on exposure until sensor saturates
- Denoted in fractions of a second:
  - 1/2000, 1/1000,...,1/250, 1/125, 1/60,...,15, 30
- Normal humans can hand-hold down to 1/60 second
  - Rule of thumb: shortest exposure = $1 / f$
    - e.g., 1/180 second for a 180*mm* lens

Adapted from Levoy (2011)

Main Side-effect of Shutter Speed

- Motion blur
- Doubling exposure time doubles motion blur

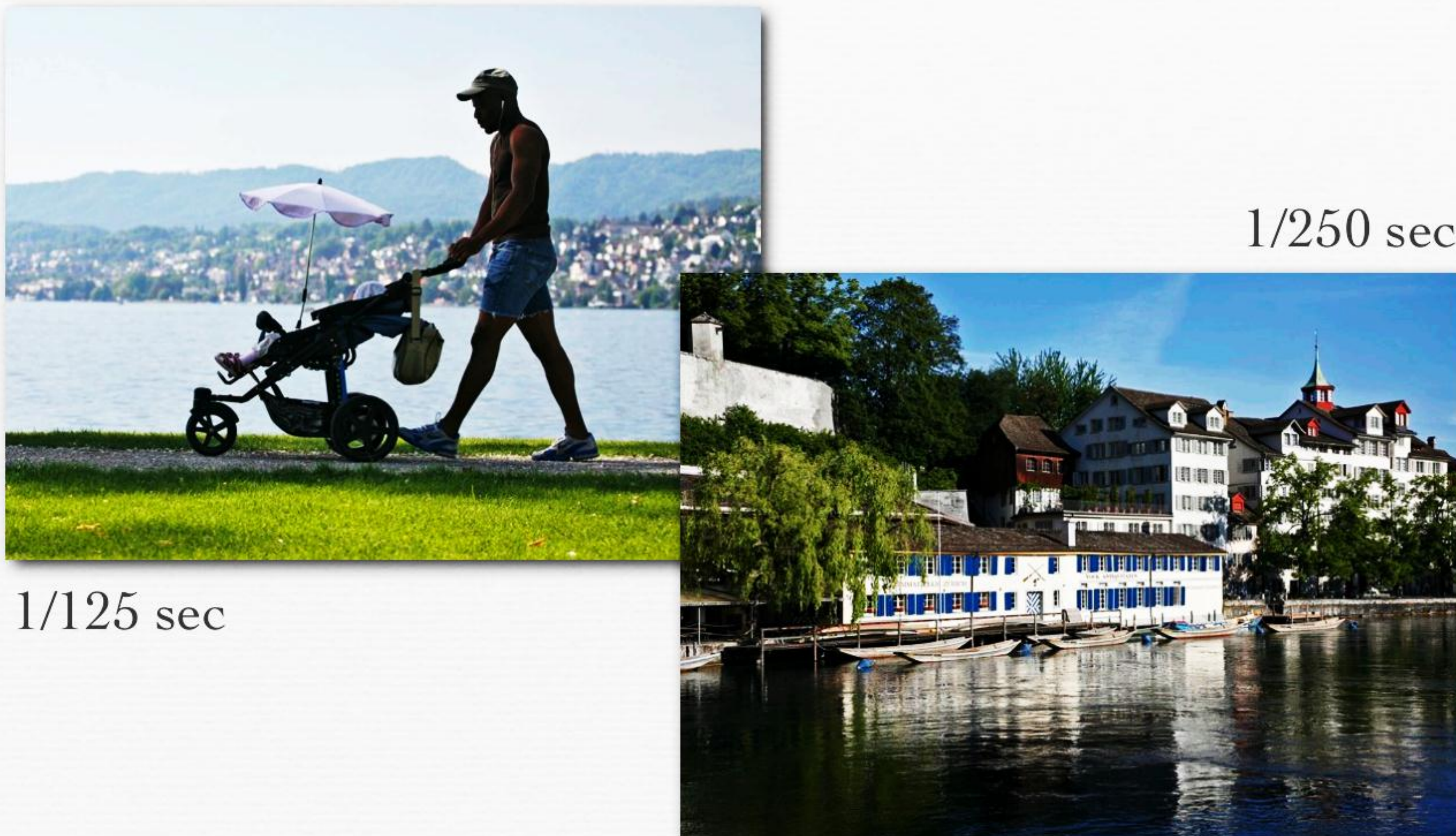


Adapted from Levoy (2011)

Useful Shutter Speeds

Adapted from Levoy (2011)

Useful Shutter Speed

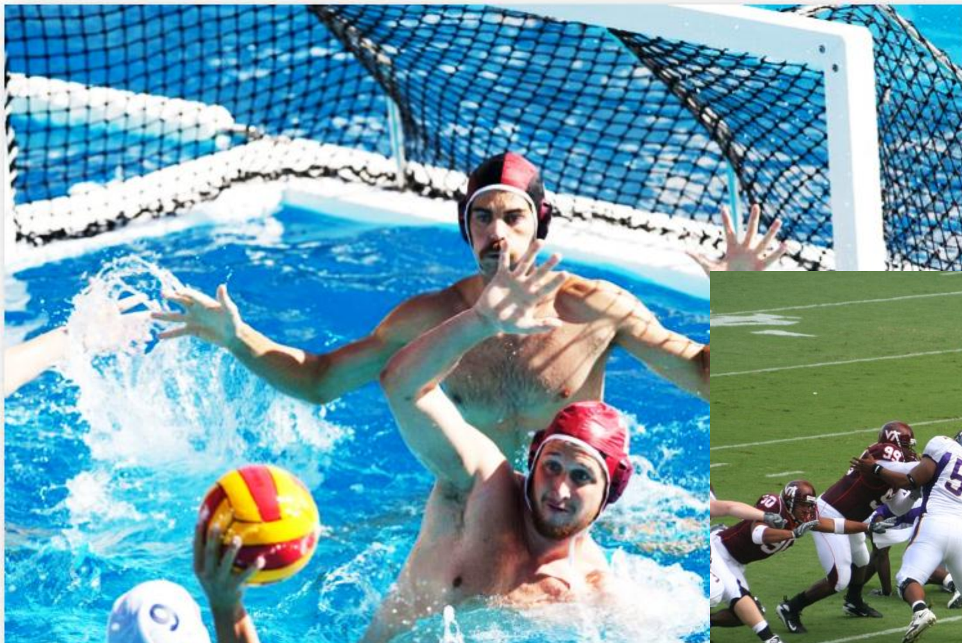


1/125 sec

1/250 sec

Adapted from Levoy (2011)

Useful Shutter Speed

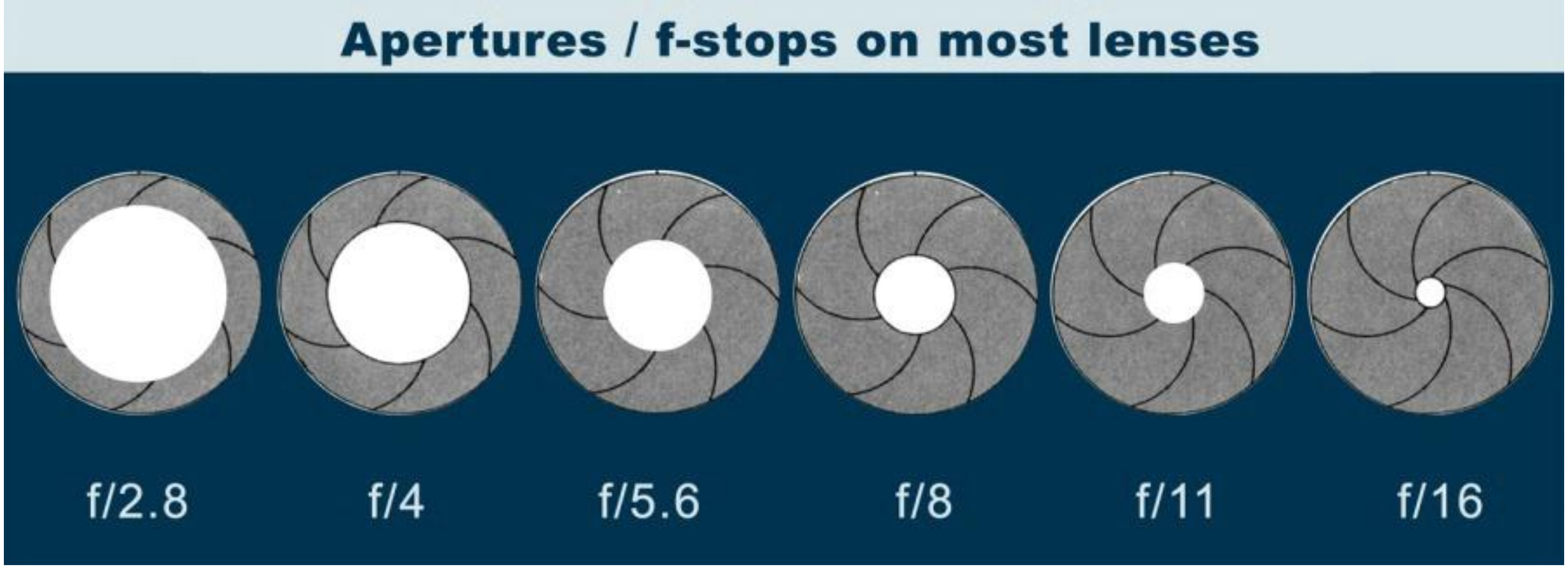


1/800 sec

1/800 sec

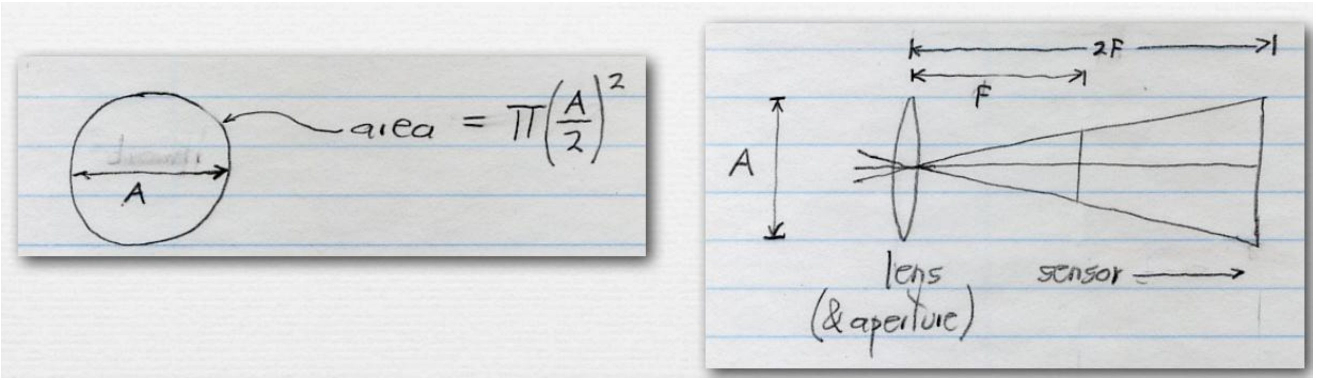
Aperture

- Irradiance on sensor is proportional to
  - Square of aperture diameter A
  - Inverse square of distance to sensor (~ focal length f)



http://photoluminary.com/wp-content/uploads/2011/03/aperture_diagram.jpg

Irradiance on Sensor



- As the diameter A of the aperture doubles, its area (hence the light that can get through it) increases by 4x.
- Think of the lens as a collection of pinholes, each having a fixed angular field of view (cone in 2nd drawing) determined by the lens design.
- A certain amount of light gets through each pinhole. By conservation of energy, that light will fall on whatever sensor is placed in its path.
- If the distance to the sensor is doubled, the area intersecting the cone increases by 4x, so the light falling per unit area decreases by 4x.

Adapted from Levoy (2011)

Aperture

- Irradiance on sensor is proportional to square of aperture diameter A
  - Inverse square of distance to sensor (~ focal length f)

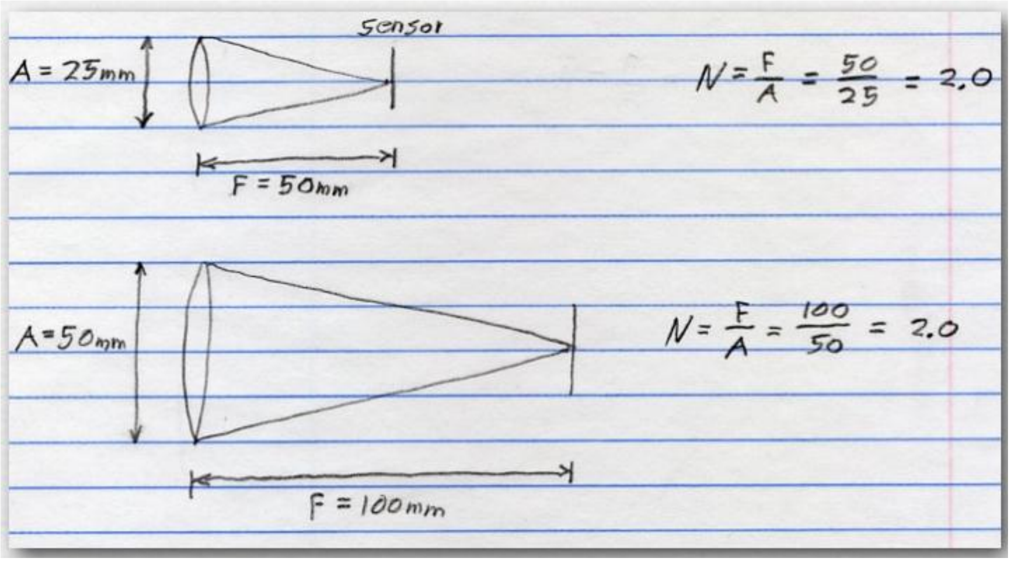
- so that aperture values give irradiance regardless of lens,
  - Aperture number N is defined relative to focal length

$$N = \frac{f}{A}$$

  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm

Adapted from Levoy (2011)

Example of F-number calculations



- A relative aperture size (called F-number or just N) of 2 is written f/2, reflecting the fact that it is computed by dividing focal length (f) by the absolute aperture diameter (A).
  - Doubling both the absolute aperture diameter (A) and the focal length (f) cancel; leaving the same relative aperture size (N).
  - In this example, both lenses are f/2.

Adapted from Levoy (2011)

Aperture

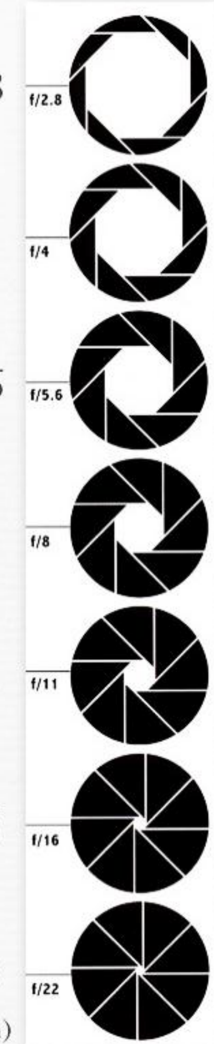
- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)
- so that aperture values give irradiance regardless of lens, *aperture number* N is defined relative to focal length

$$N = \frac{f}{A}$$

  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm

  ∴ low F-number (N) on long zooms require fat lenses
- doubling N reduces A by 2×, hence light by 4×
  - going from f/2.0 to f/4.0 cuts light by 4×
  - to cut light by 2×, increase N by $\sqrt{2}$

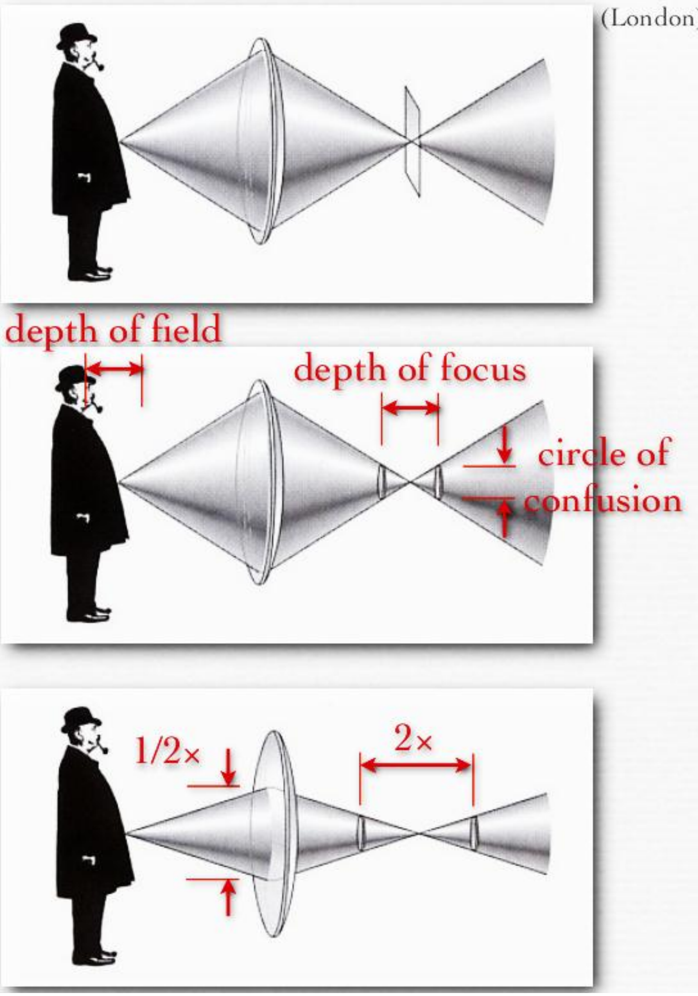
f/2.8
f/4
f/5.6
f/8
f/11
f/16
f/22
(London)



Adapted from Levoy (2011)

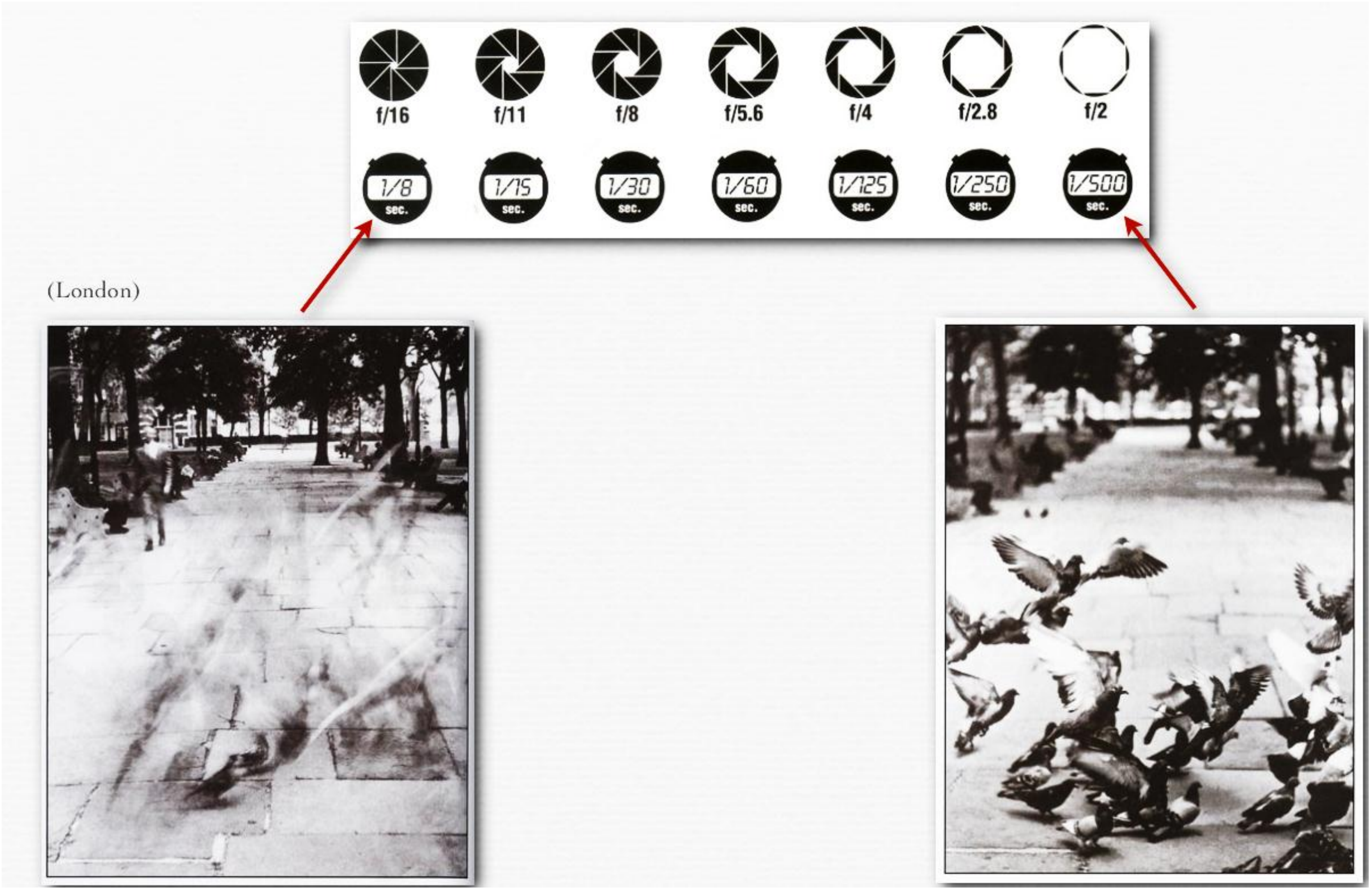
Depth of Field

- a point in the scene is focused at a point on the sensor
- if we move the sensor in depth, the point becomes blurred
- if it blurs too much, it exceeds our allowable *circle of confusion*
- the depth where this happens is called the *depth of focus*
- this corresponds in the scene to a *depth of field*
- halving the aperture diameter doubles the depth of field



Adapted from Levoy (2011)

Trading off motion blur and depth of field

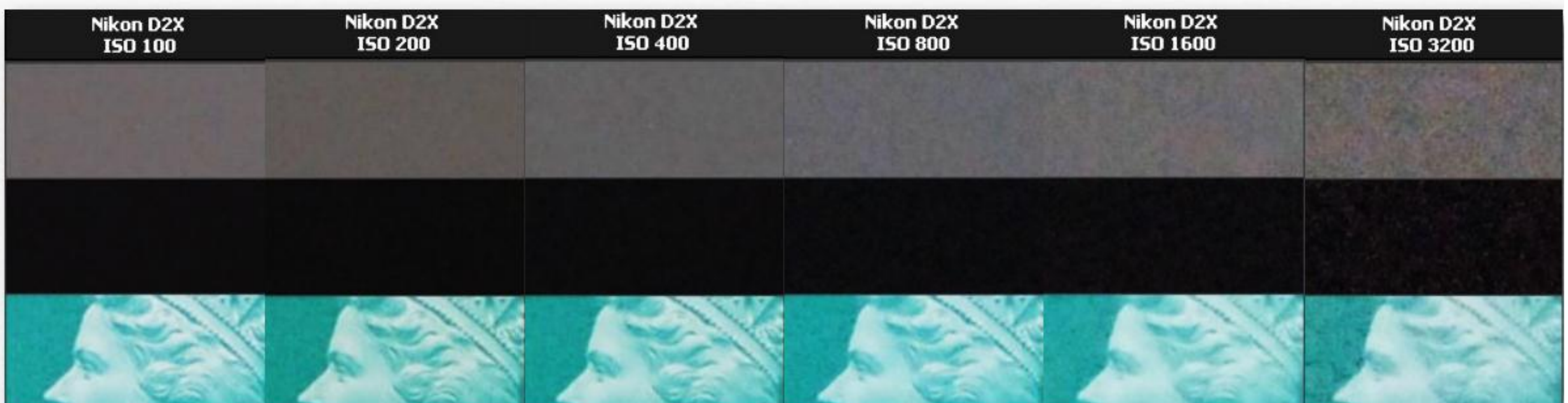


Adapted from Levoy (2011)

Sensitivity (ISO)

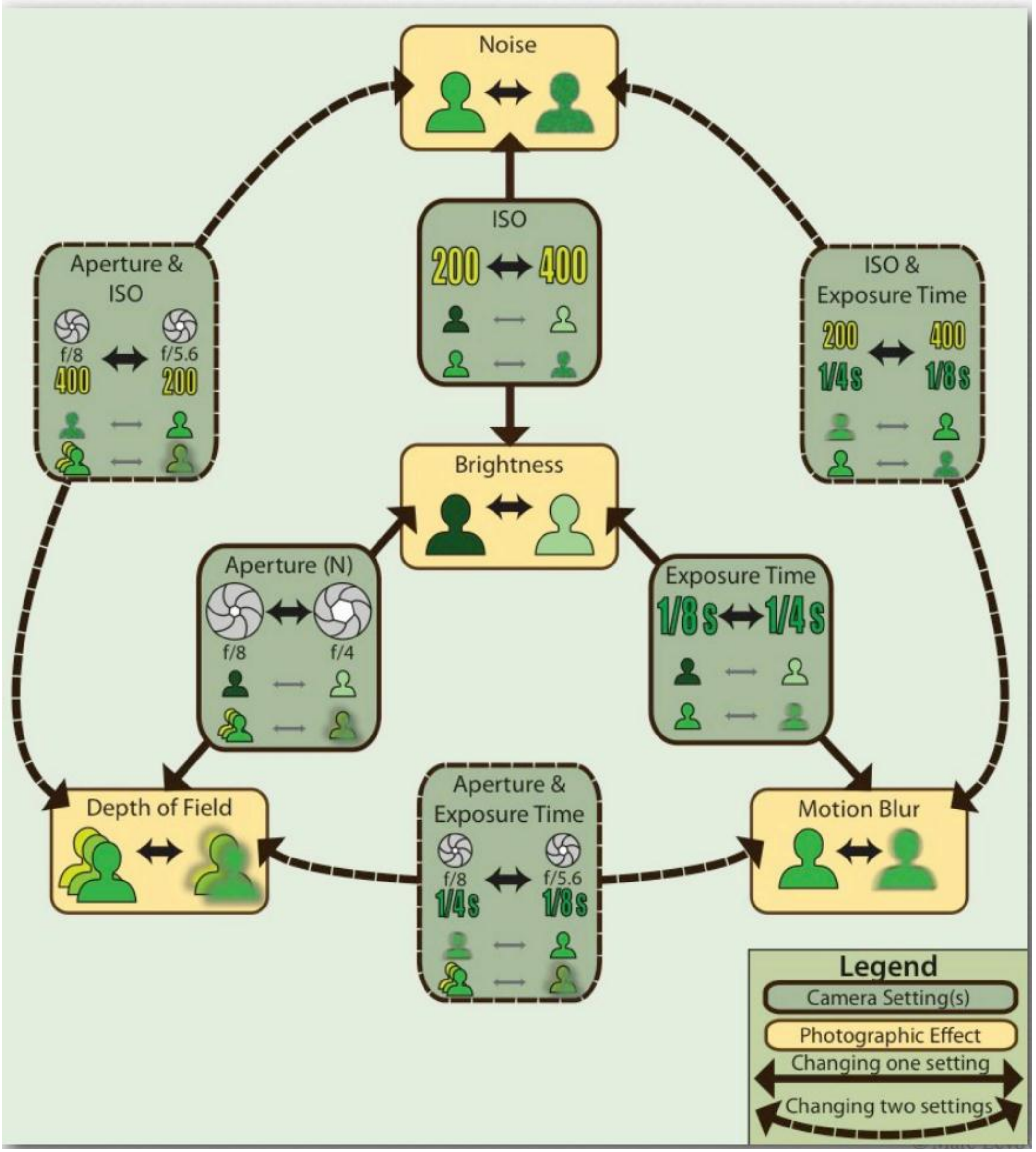
- third variable for exposure
- film: trade sensitivity for grain
- digital: trade sensitivity for noise
  - multiply signal before analog-to-digital conversion
  - linear effect (200 ISO needs half the light as 100 ISO)

more in noise lecture



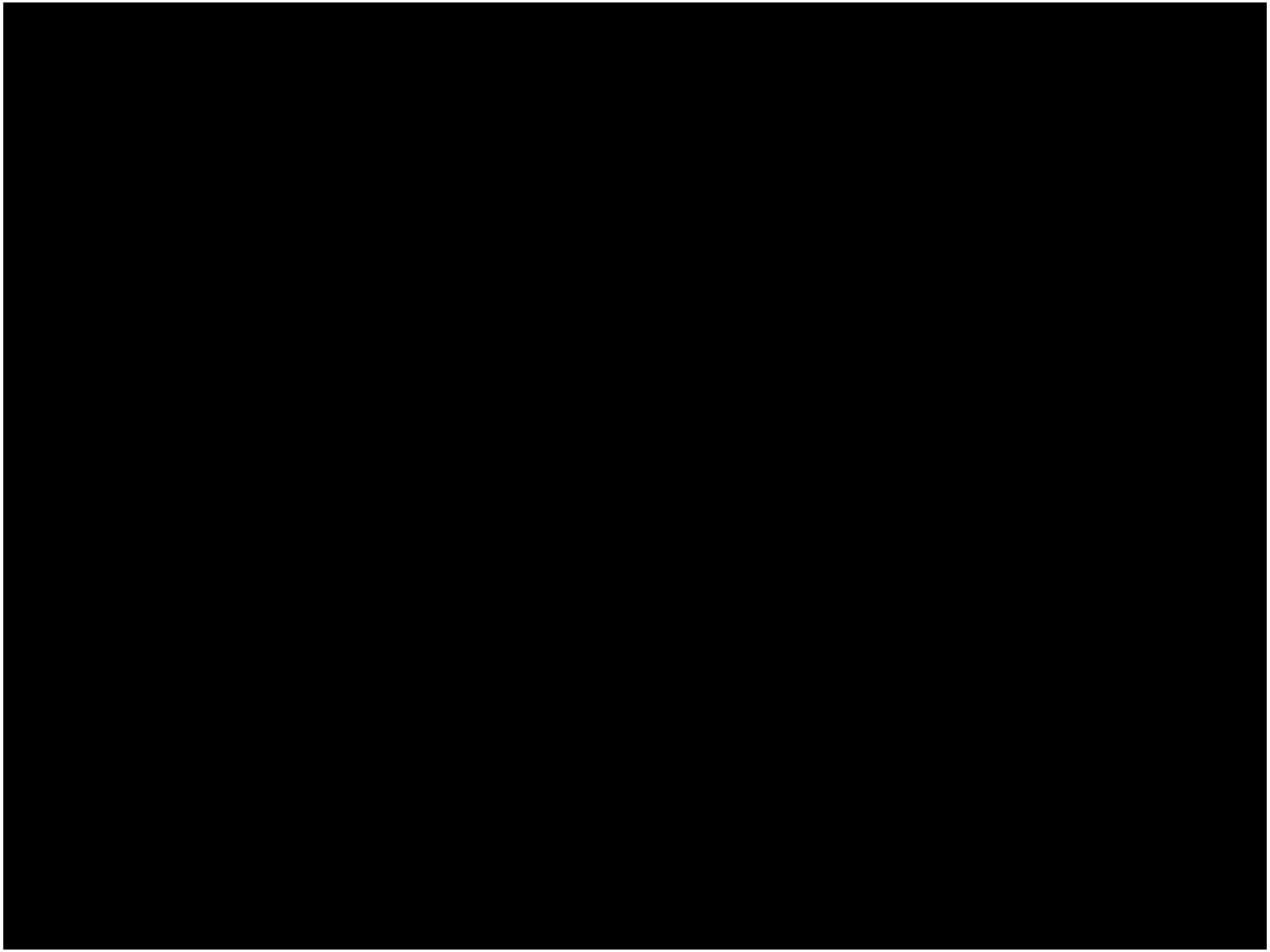
Adapted from Levoy (2011)

Tradeoff Affecting Brightness



Adapted from Levoy (2011)

Canon EOS C300



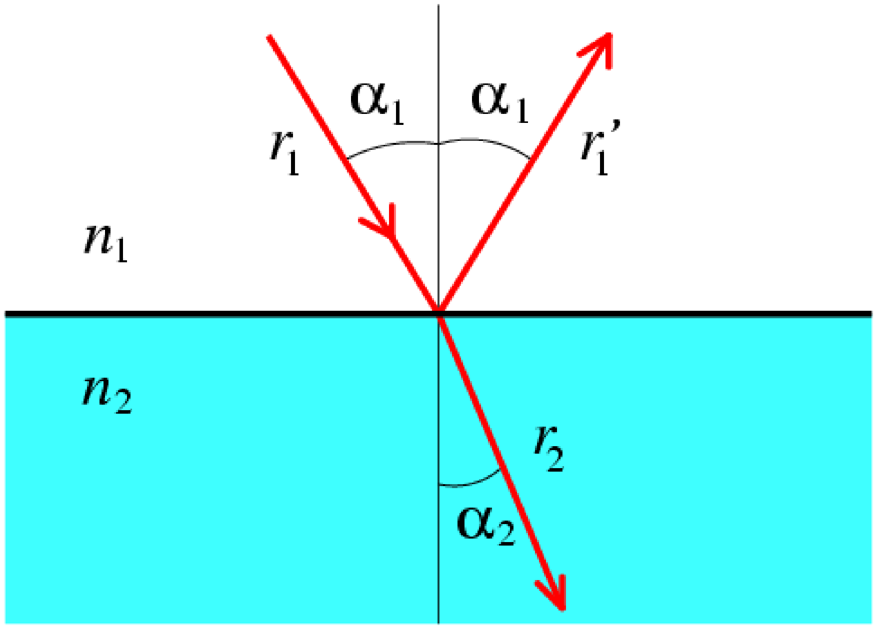
Canon EOS C300

Cameras & Lenses

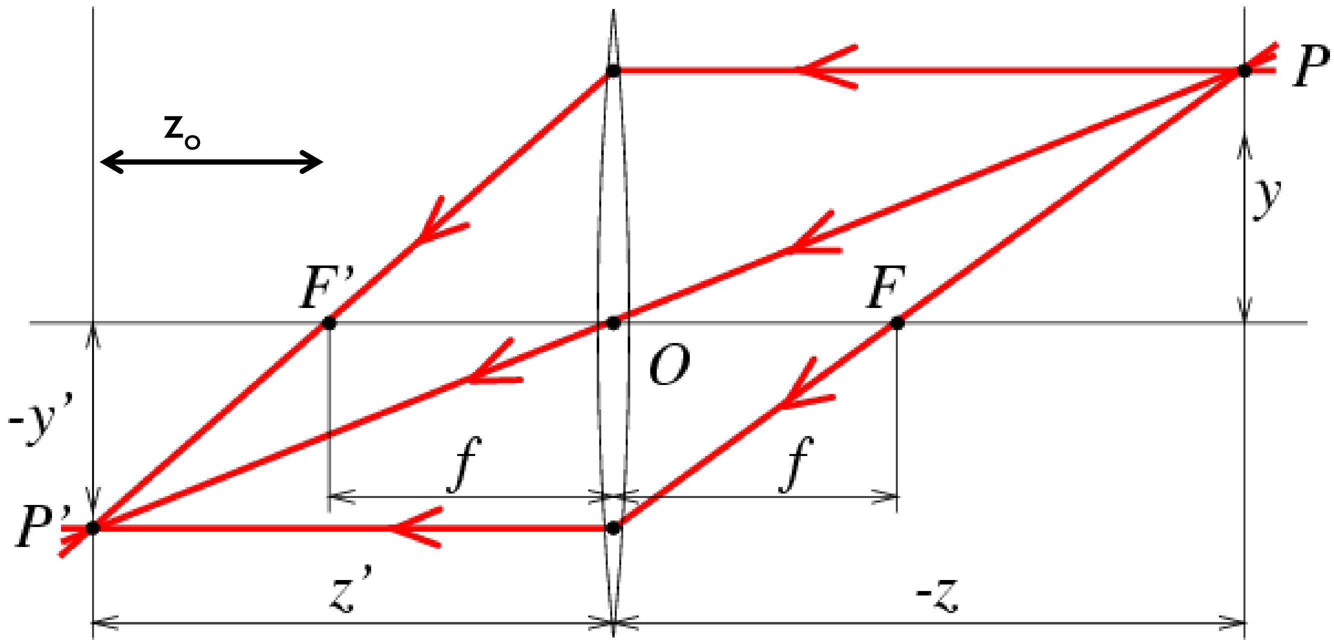
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction



Thin Lenses



Snell's law:

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

➡

Small angles:

$$n_1 \alpha_1 \approx n_2 \alpha_2$$

$n_1 = n$ (lens)

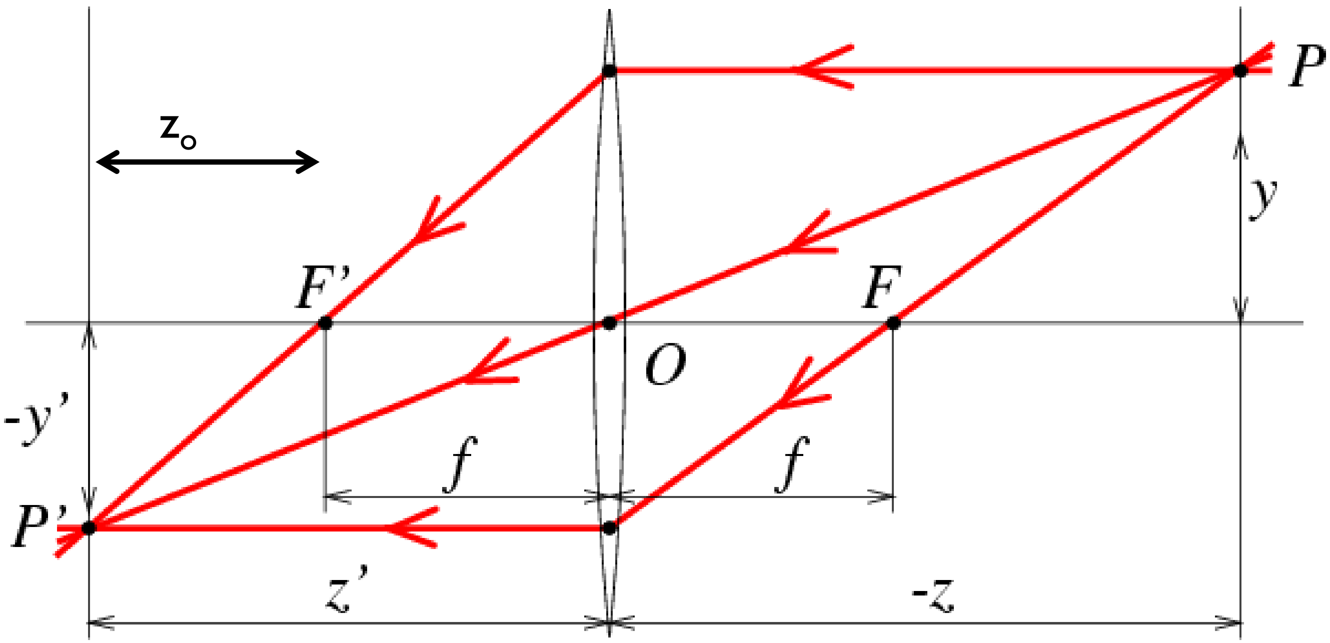
$n_1 = 1$ (air)

➡

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$f = \frac{R}{2(n-1)}$$

Thin Lenses



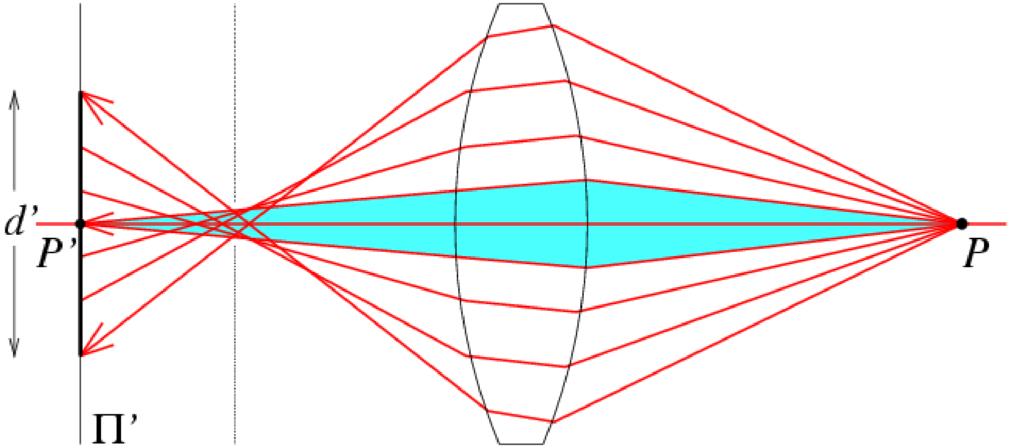
$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases}$$

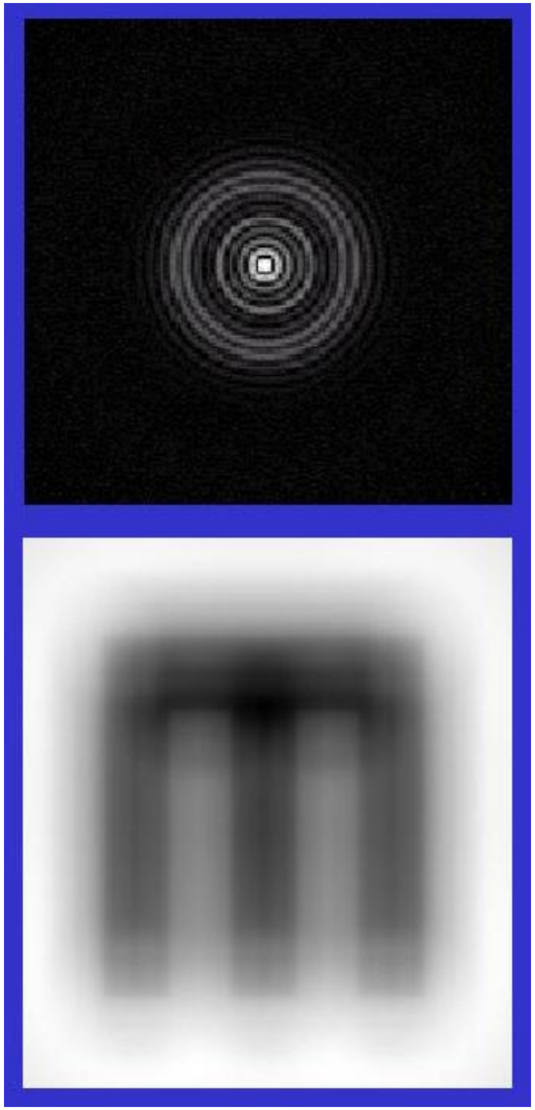
$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Issues with lenses: Spherical aberration

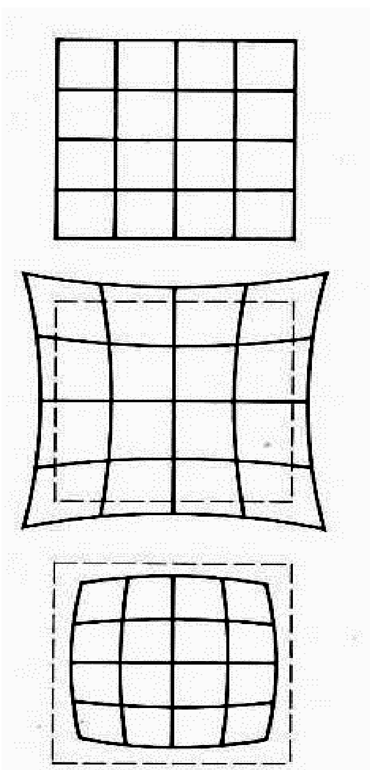
- Rays farther from the optical axis focus closer





Issues with lenses: Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

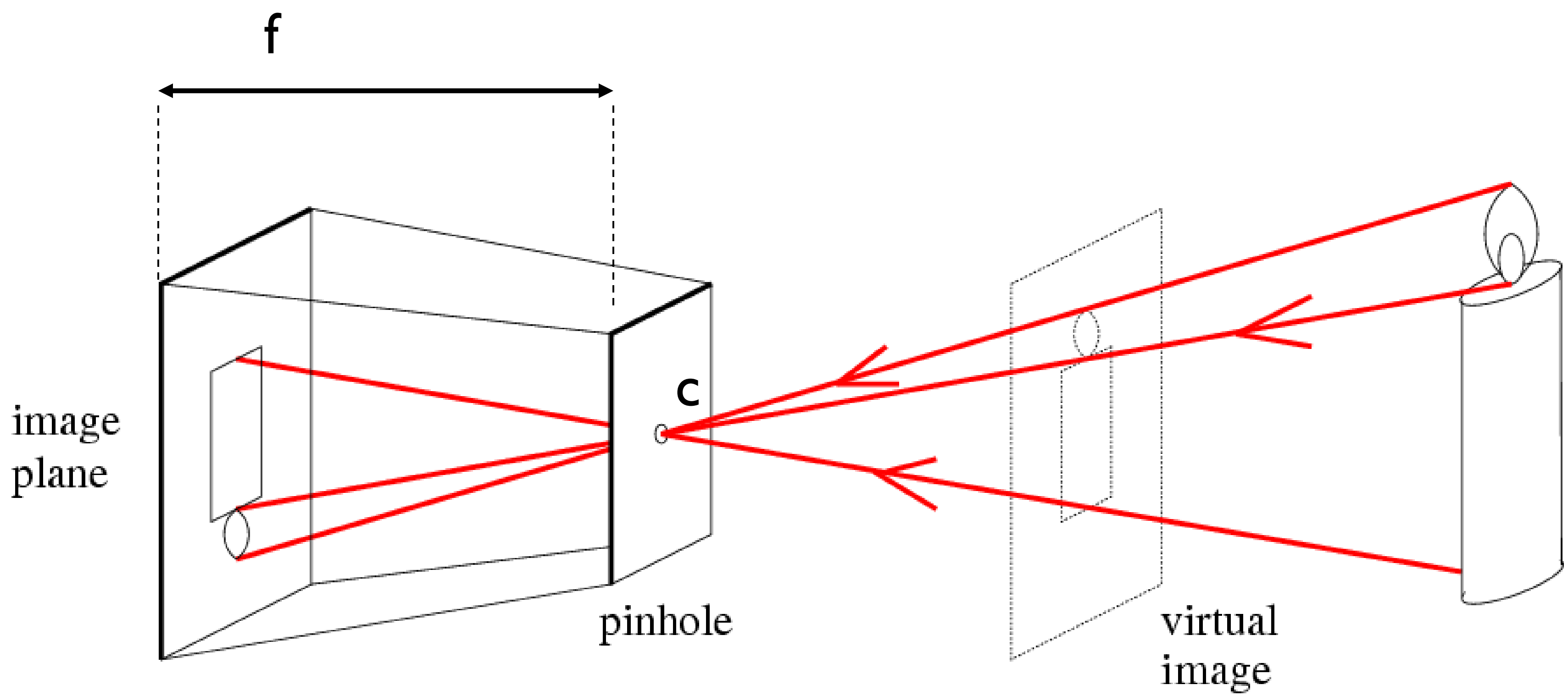
Barrel

Outline

Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Pinhole camera



$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

f = focal length

c = center of the camera

$$\Re^3 \xrightarrow{E} \Re^2$$

$$(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})$$

Is this a linear transformation?

No — division by z is nonlinear

How to make it linear?

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection Transformation

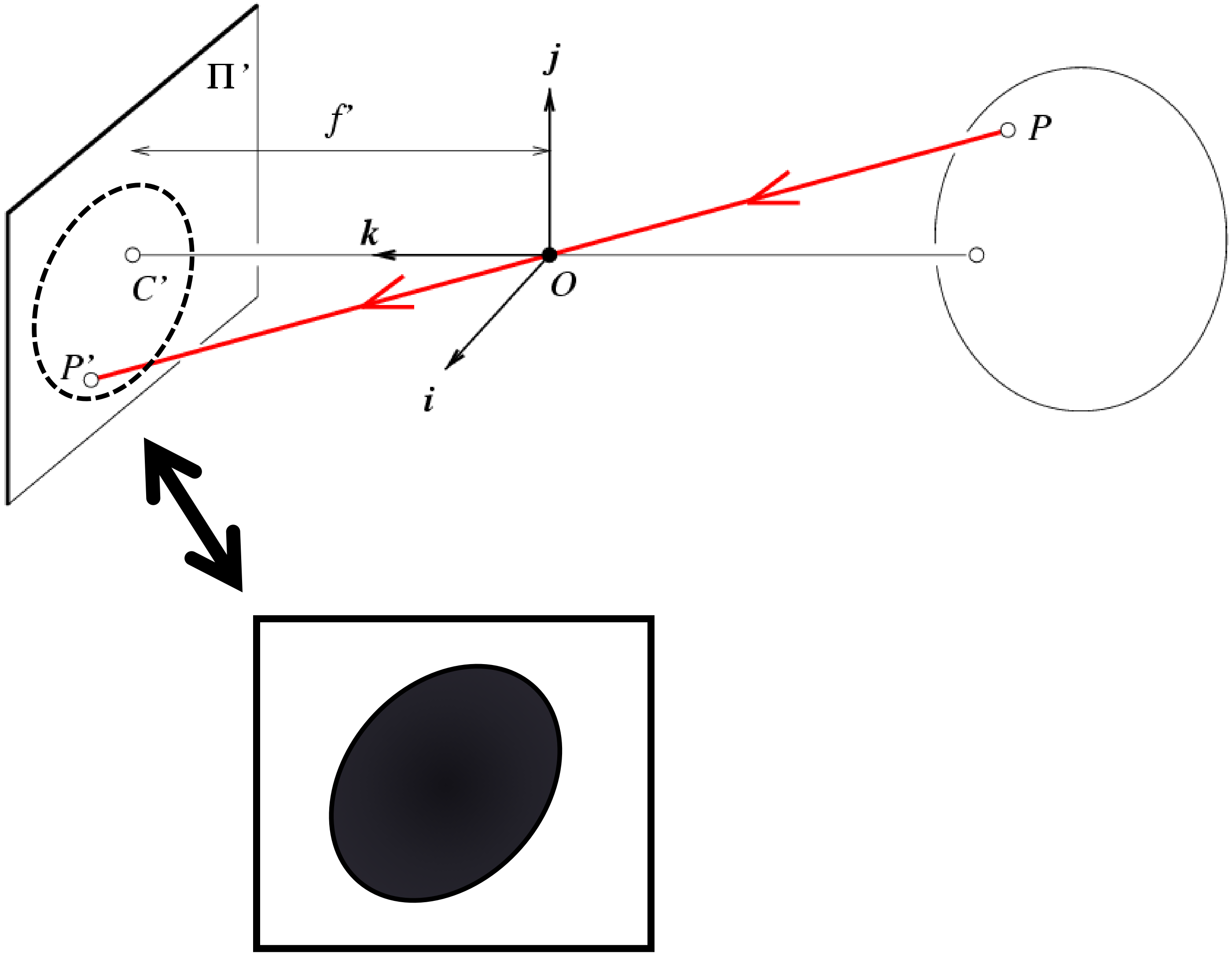
$$X' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$X' = M\;X$$

$$\Re^4 \xrightarrow{H} \Re^3$$

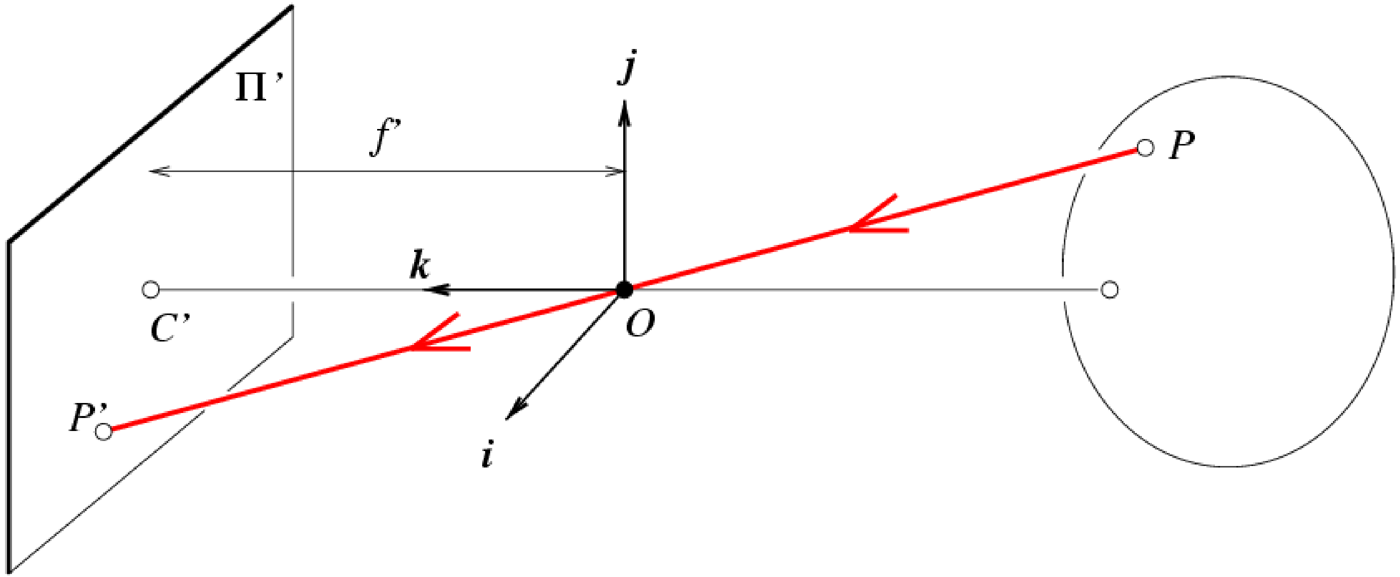
$$X_i' = \begin{bmatrix} f\,\dfrac{x}{z} \\ f\,\dfrac{y}{z} \end{bmatrix}$$

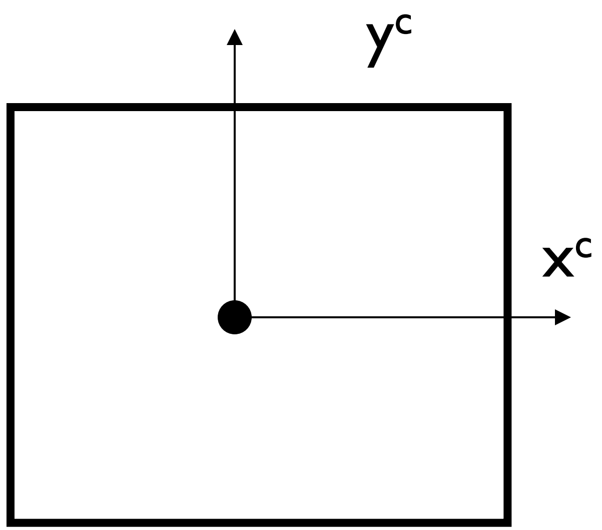
From retina plane to images



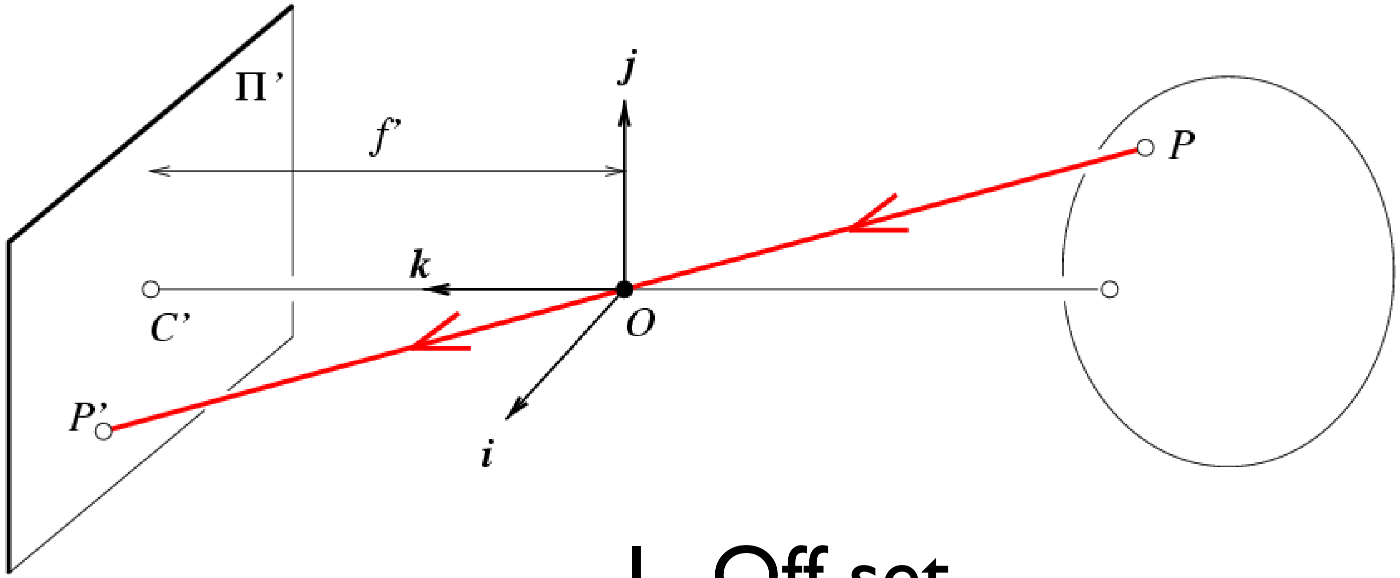
Pixels, bottom-left coordinate systems

Coordinate systems

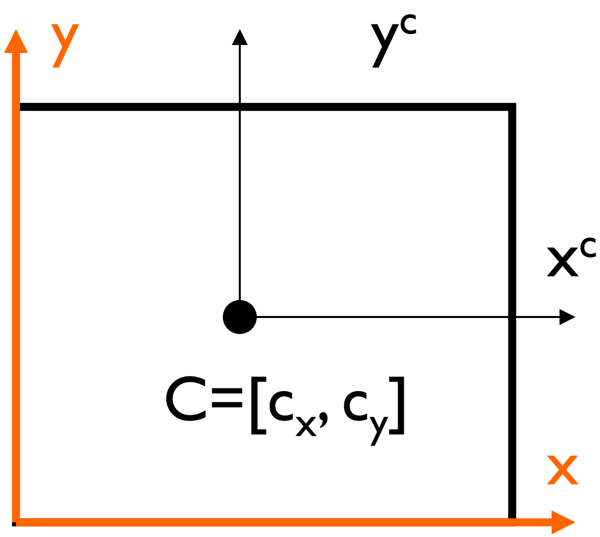




Converting to pixels

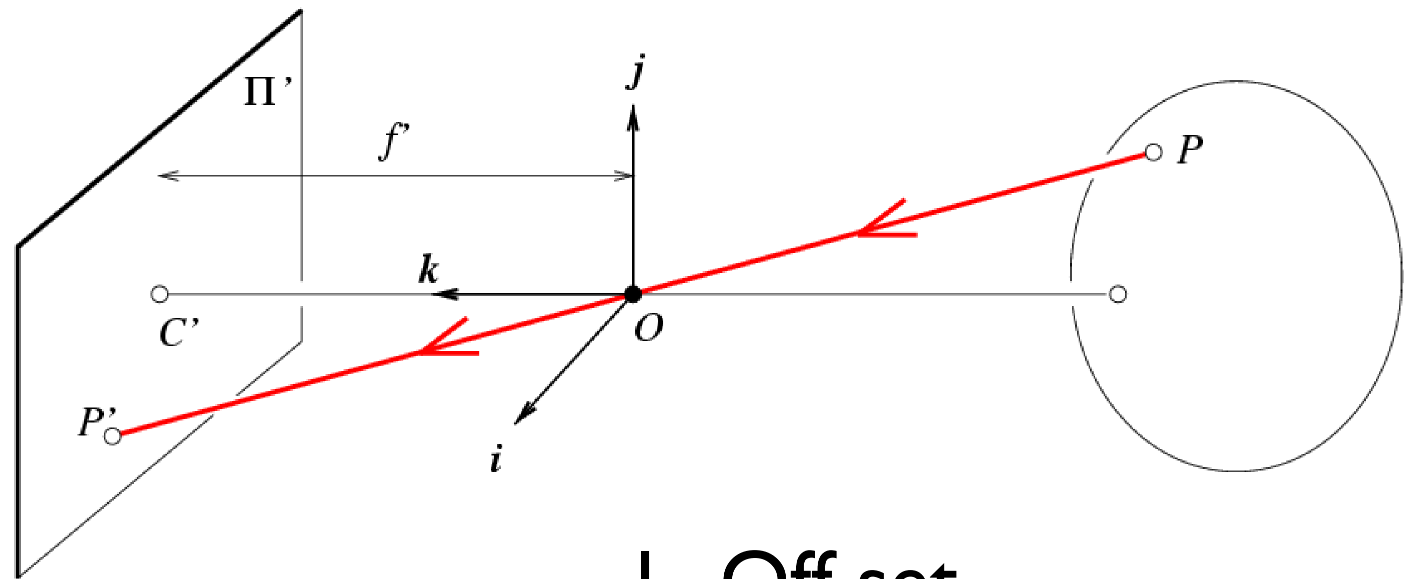


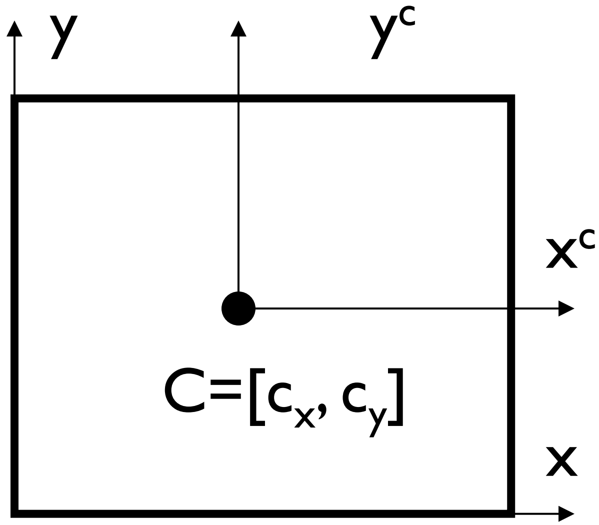
1. Off set



$$(x, y, z) \rightarrow \left(f\frac{x}{z} + c_x,\ f\frac{y}{z} + c_y\right)$$

Converting to pixels





1. Off set
2. From metric to pixels

$$(x, y, z) \rightarrow (\underbrace{f\,k}_{\alpha}\frac{x}{z} + c_x, \underbrace{f\,l}_{\beta}\frac{y}{z} + c_y)$$

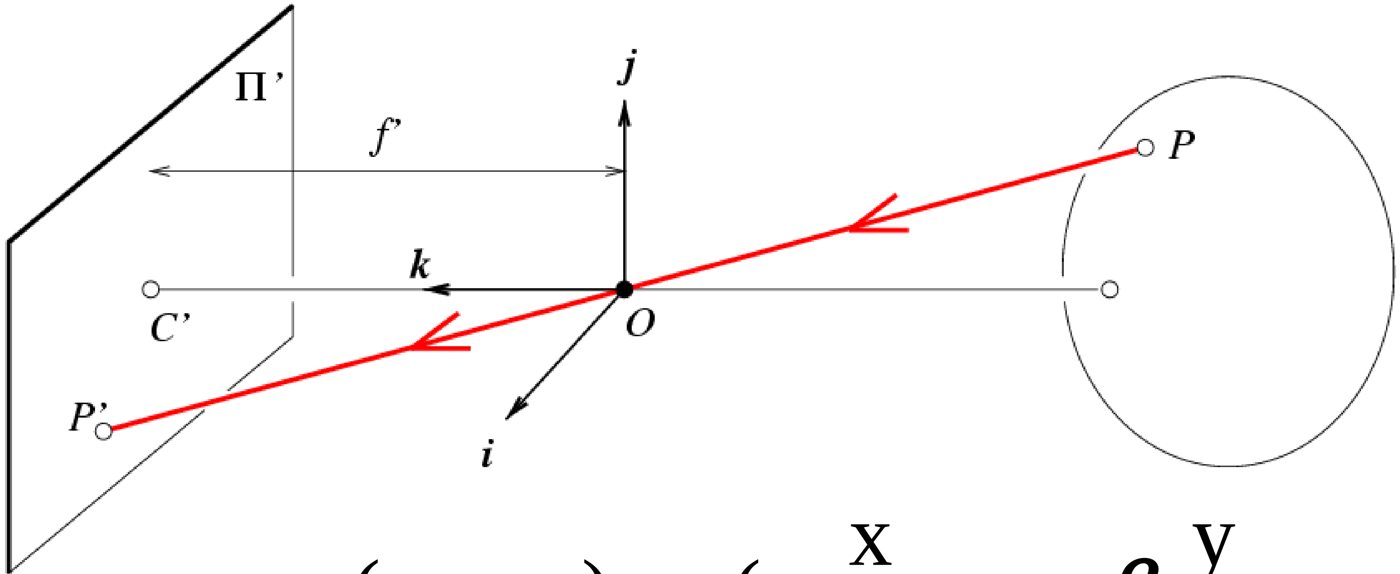
Units: k,l : pixel/m

f : m

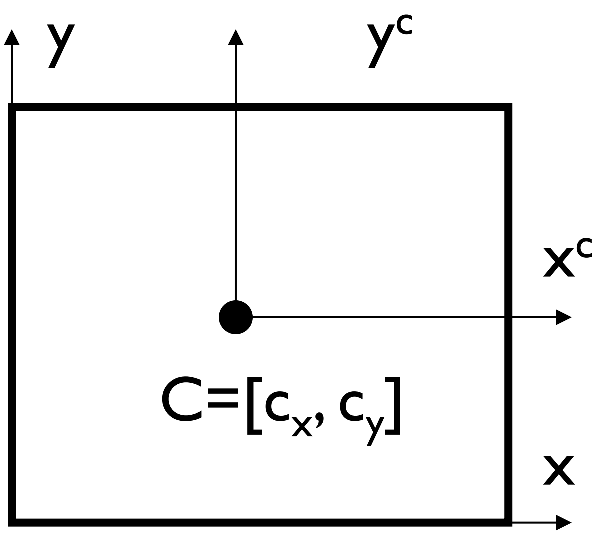
Non-square pixels

α, β : pixel

Converting to pixels

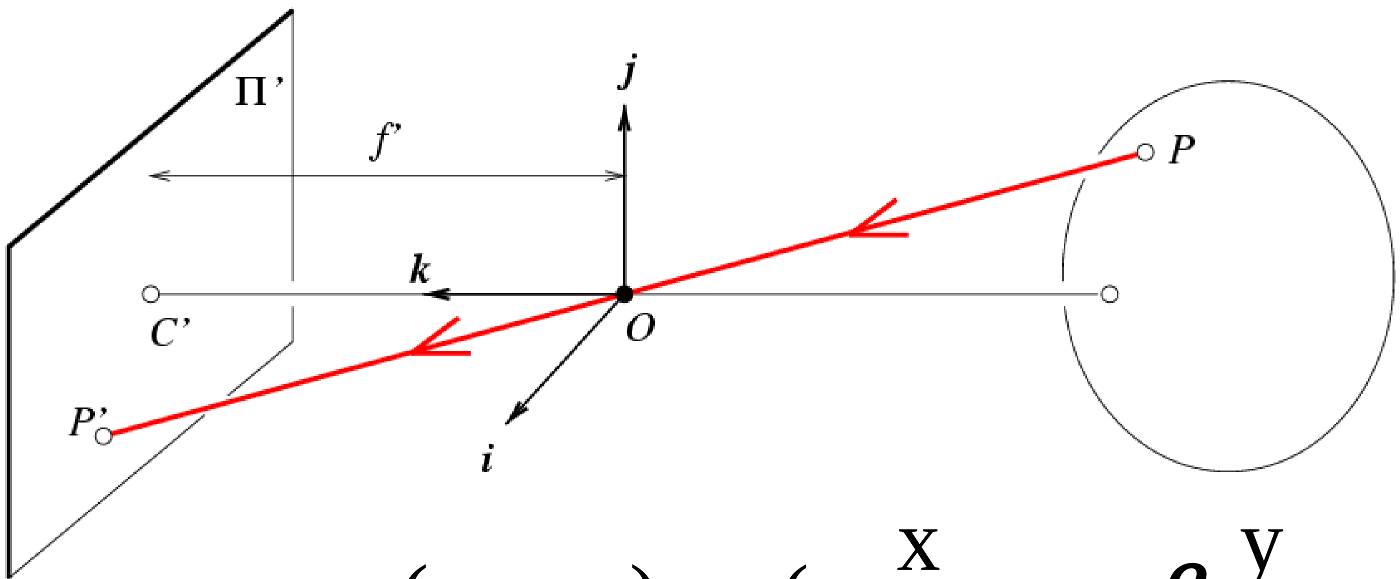


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$

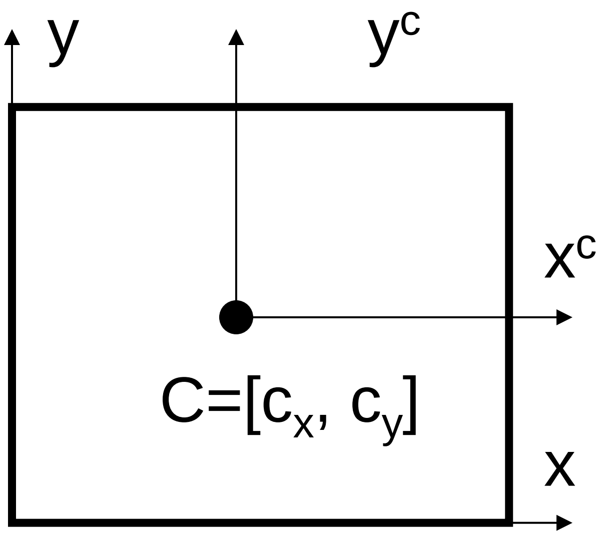


Matrix form?

Camera Matrix

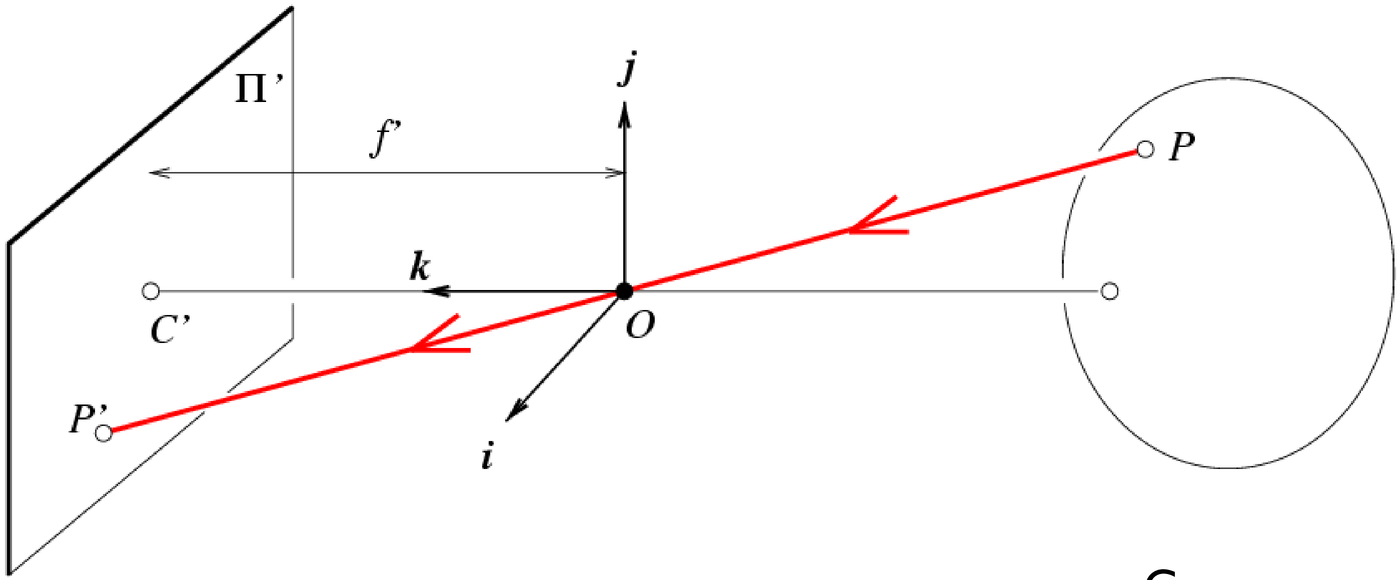


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$



$$X' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera Matrix



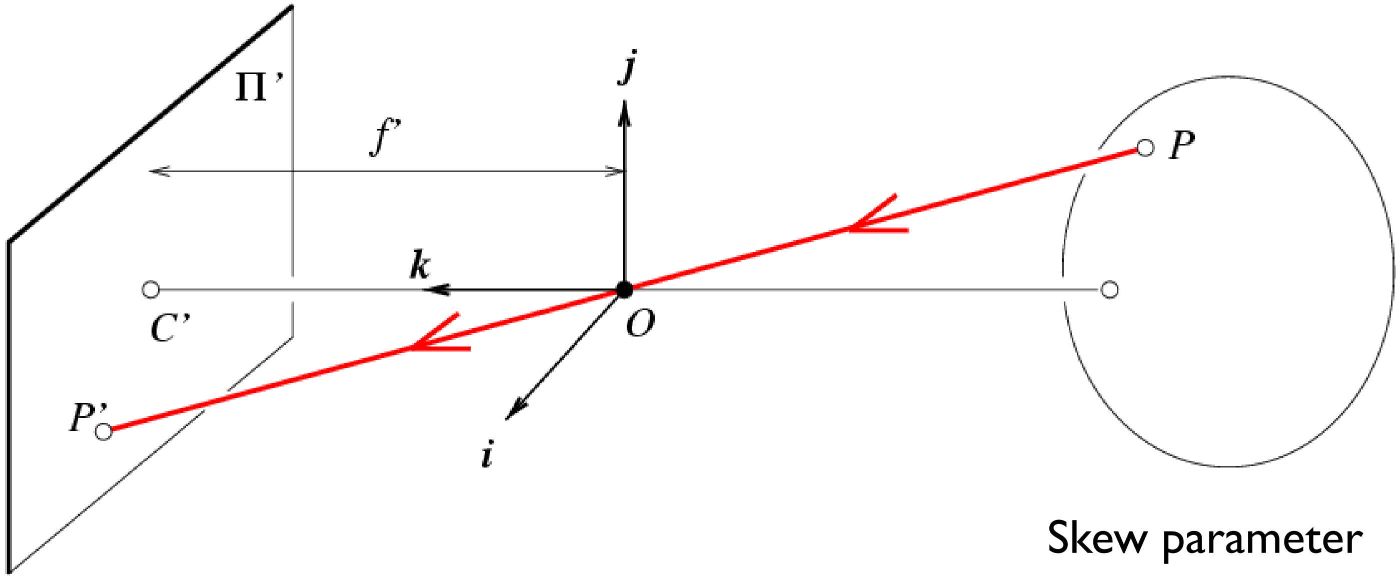
Camera matrix K

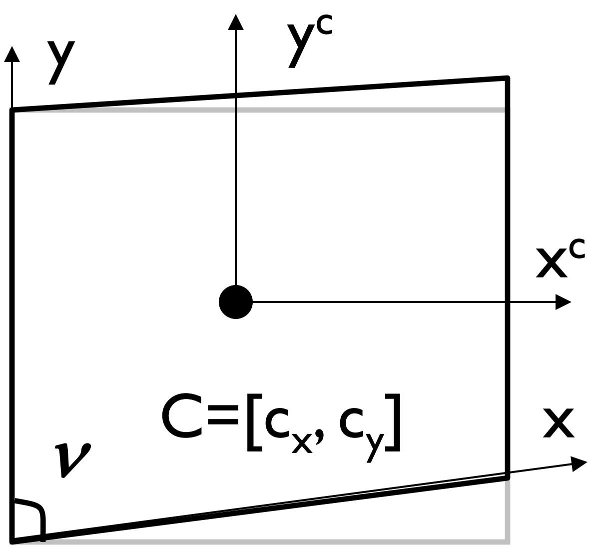
$$X' = M\,X$$

$$= K\begin{bmatrix} I & 0 \end{bmatrix} X$$

$$X' = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Finite projective cameras



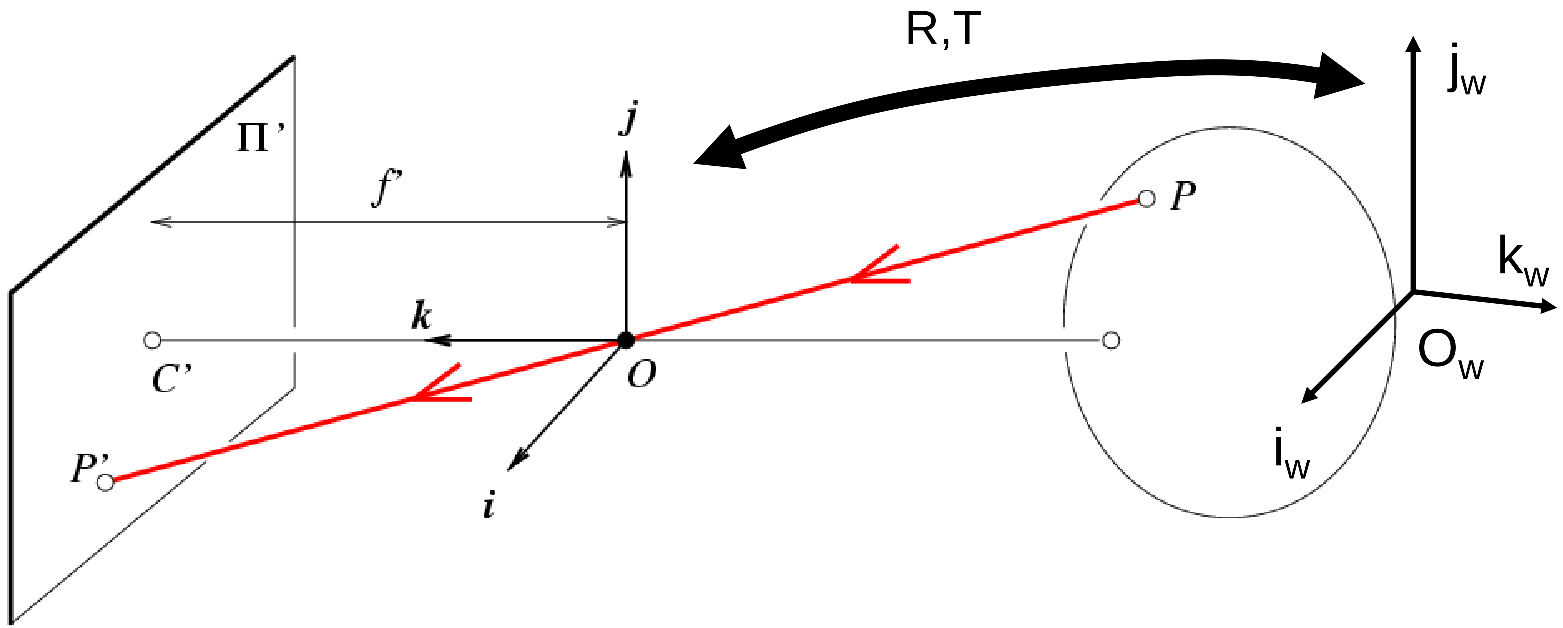


Skew parameter

$$X' = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

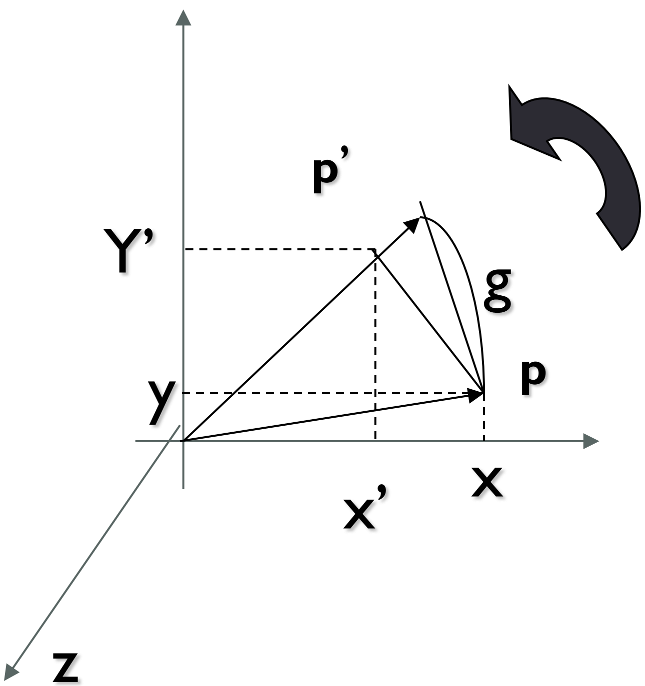
K has 5 degrees of freedom!

World reference system



3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

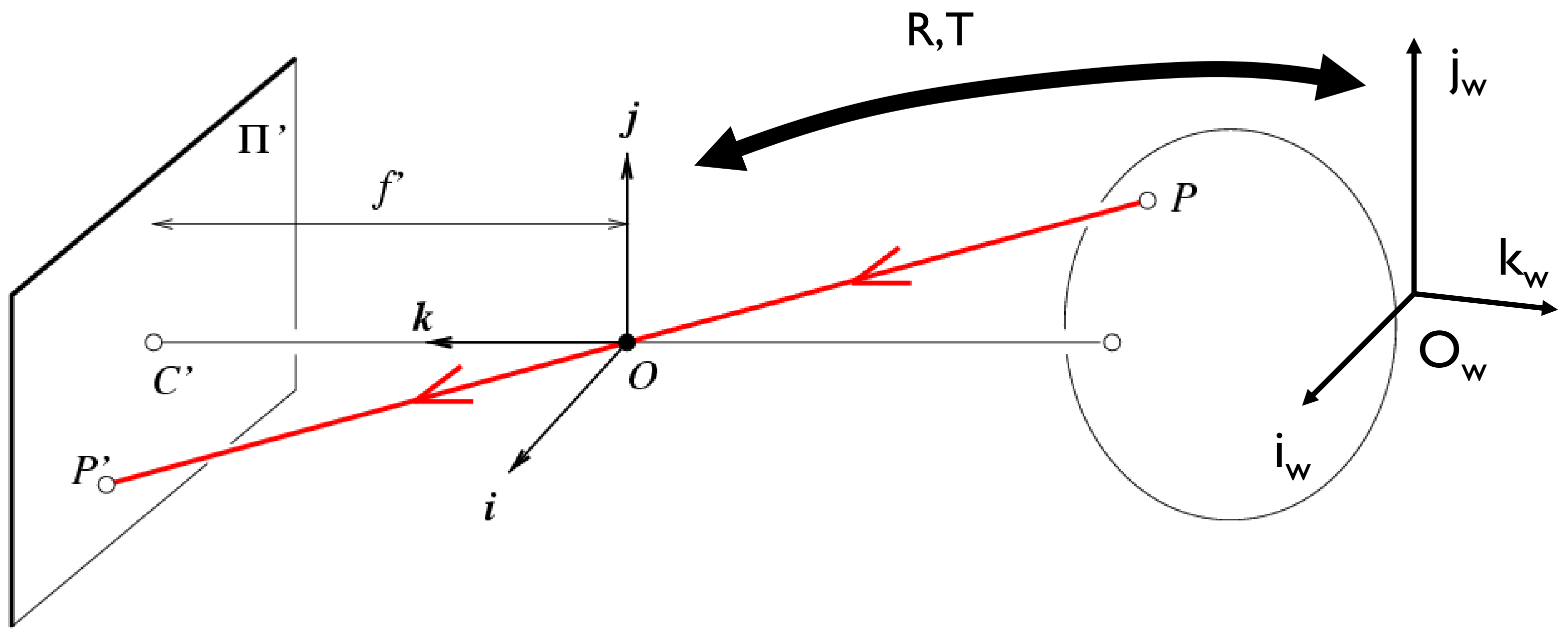


$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

World reference system



In 4D homogeneous coordinates:

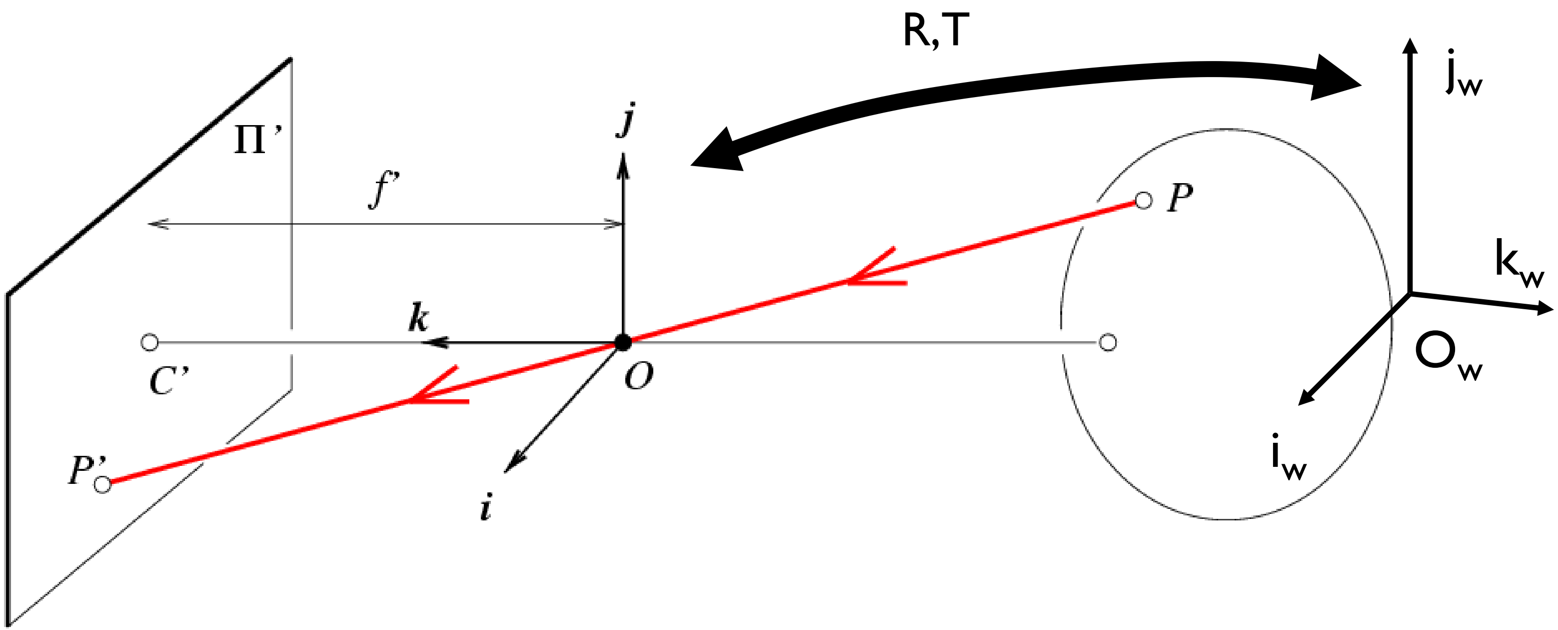
$$X = \begin{bmatrix} R & T \end{bmatrix} X_w$$

$$X' = M\, X_w = K \begin{bmatrix} R & T \end{bmatrix} X_w$$

Internal parameters

External parameters

Projective cameras



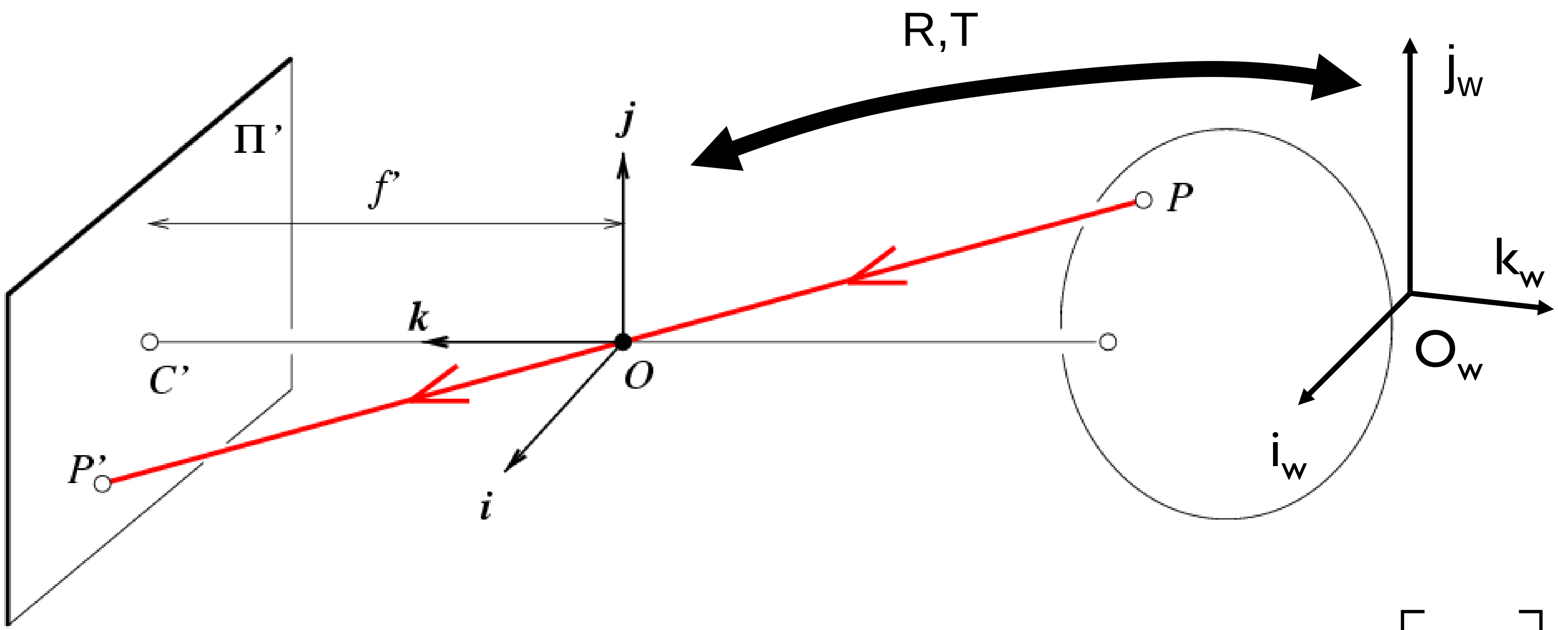
$$X'_{3\times1} = M_{3x4}\ X_w \quad = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} X_{w\,4\times1}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

Projective cameras



$$X'_{3\times1} = M\,X_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} X_{w\,4\times1} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$(x, y, z)_w \to \left(\frac{\mathbf{m}_1 X_w}{\mathbf{m}_3 X_w}, \frac{\mathbf{m}_2 X_w}{\mathbf{m}_3 X_w}\right)$$

M is defined up to scale!

Multiplying M by a scalar won't change the image

Theorem (Faugeras, 1993)

$$M = K[R \quad T] = [KR \quad KT] = [A \quad b]$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{aligned} \alpha &= f\ k; \\ \beta &= f\ l \end{aligned} \qquad A = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix}$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

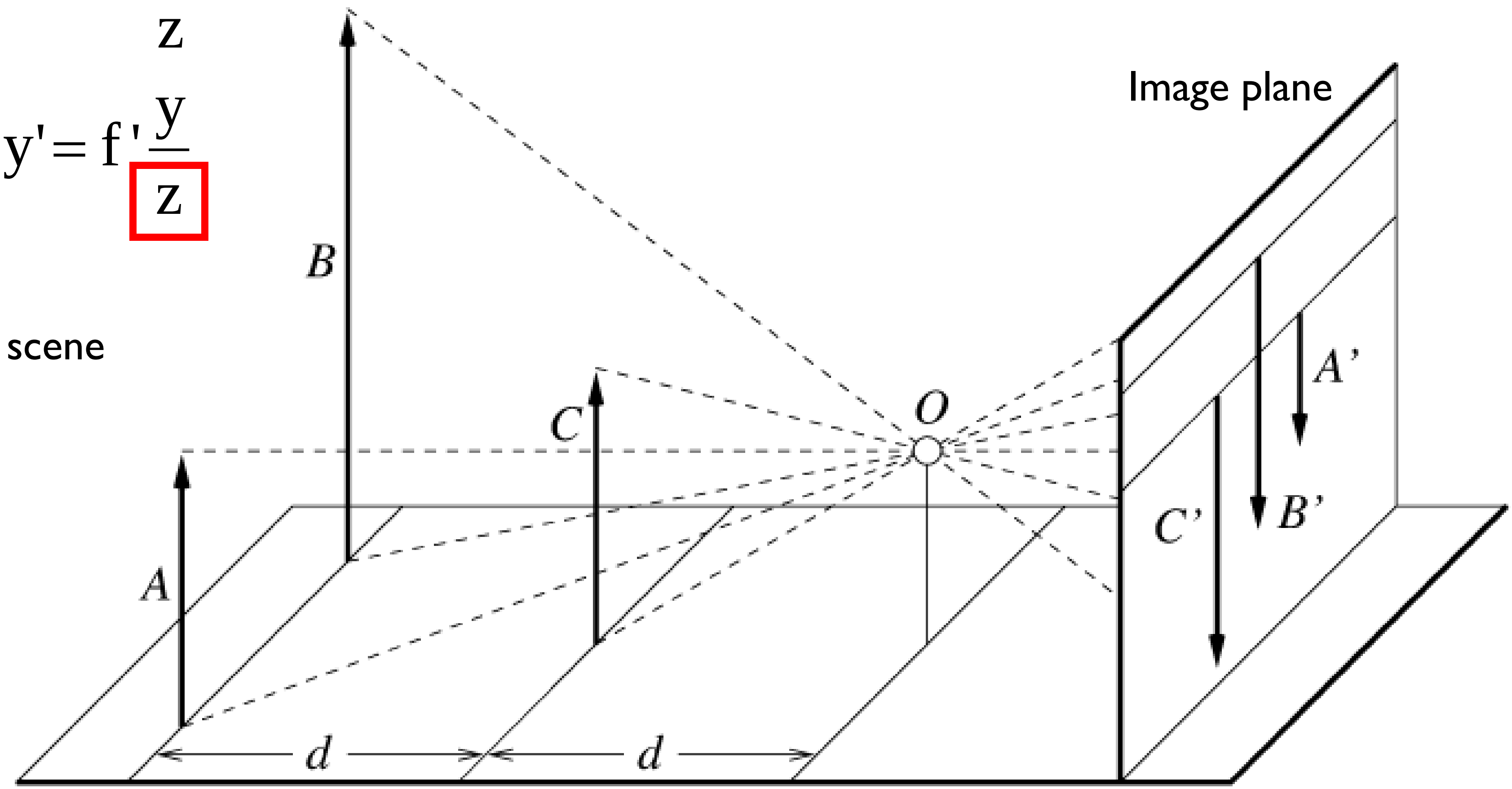
Properties of Projection

- Points project to points
- Lines project to lines

Properties of Projection

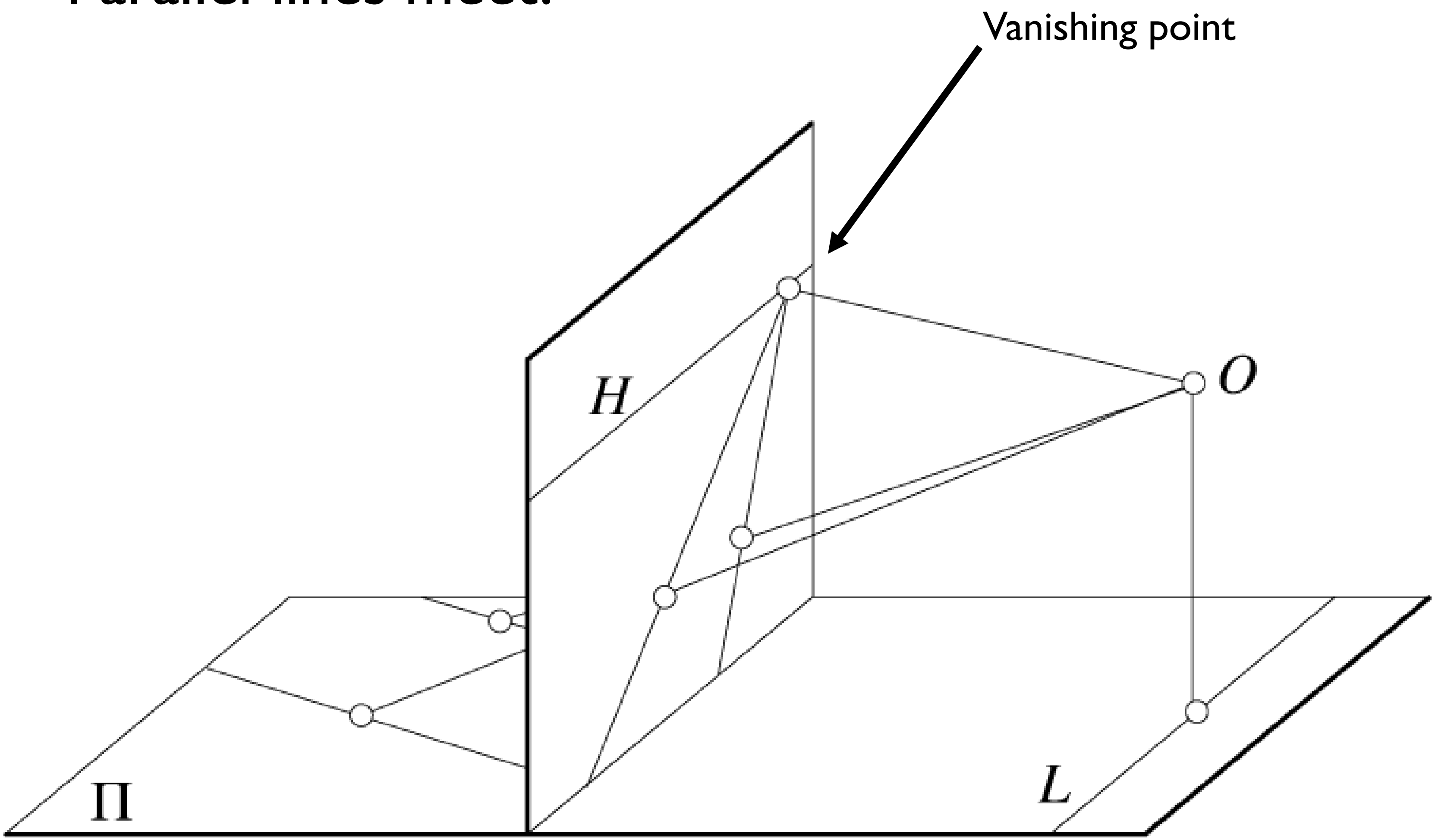
• Distant objects look smaller

$$\begin{cases} x' = f' \dfrac{x}{z} \\ y' = f' \dfrac{y}{z} \end{cases}$$



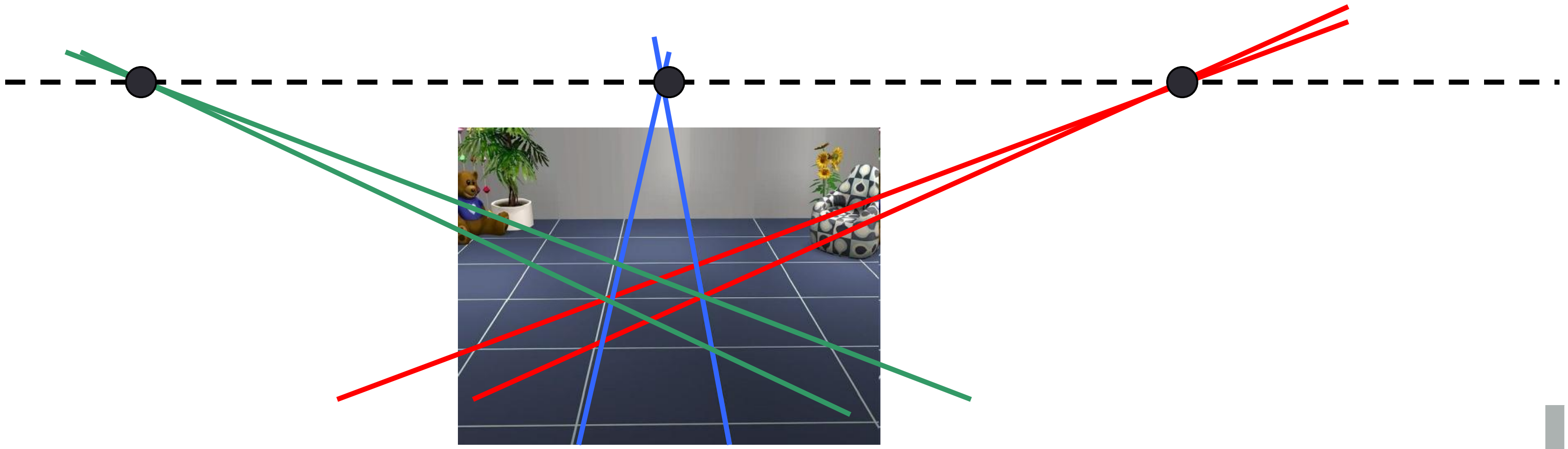
Properties of Projection

- Angles are not preserved
- Parallel lines meet!

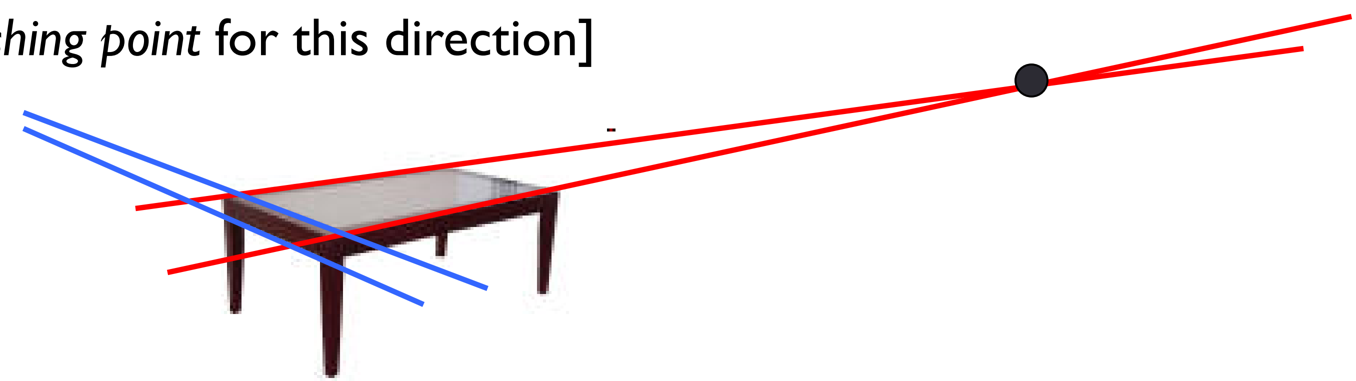


Vanishing points

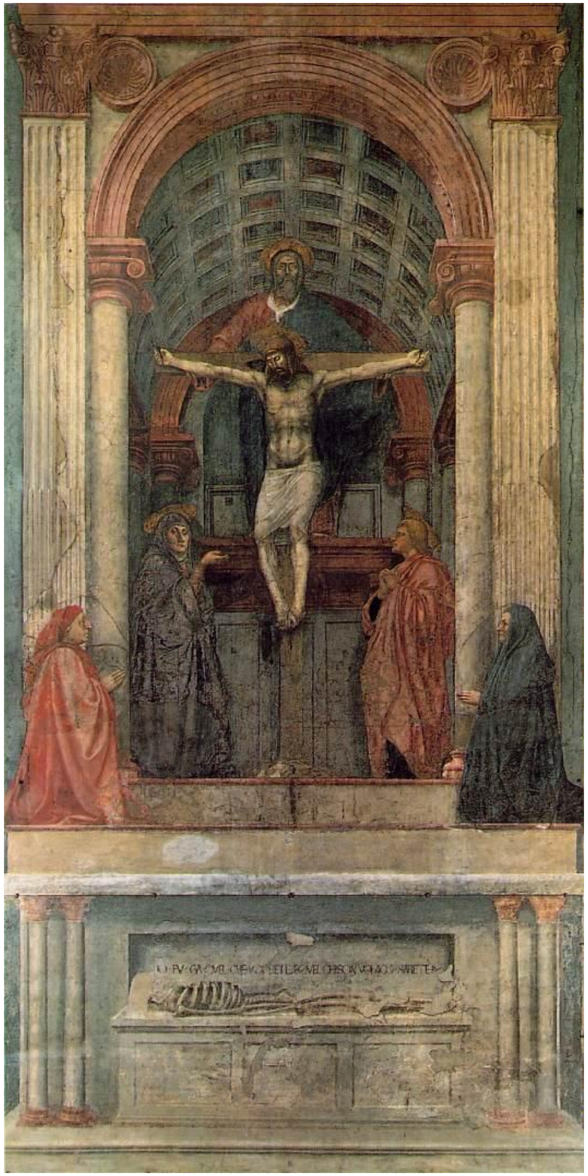
•Sets of parallel lines on the same plane lead to *collinear* vanishing points [The line is called the *horizon* for that plane]

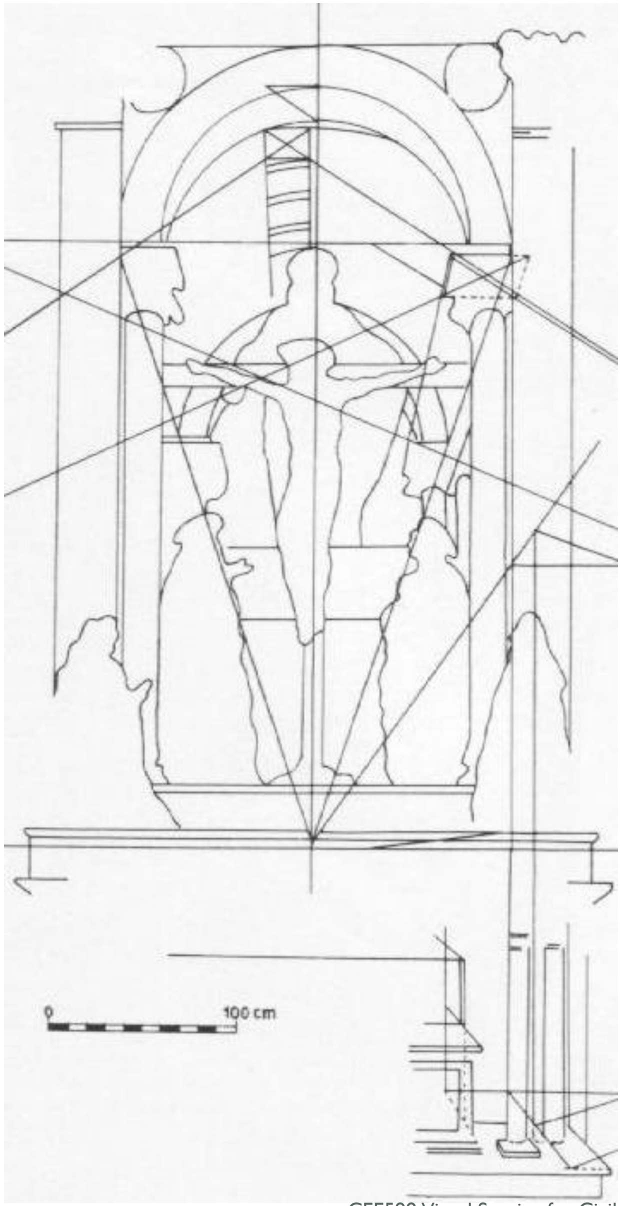


- Each set of parallel lines meets at a different point
  [The *vanishing point* for this direction]



One-point perspective



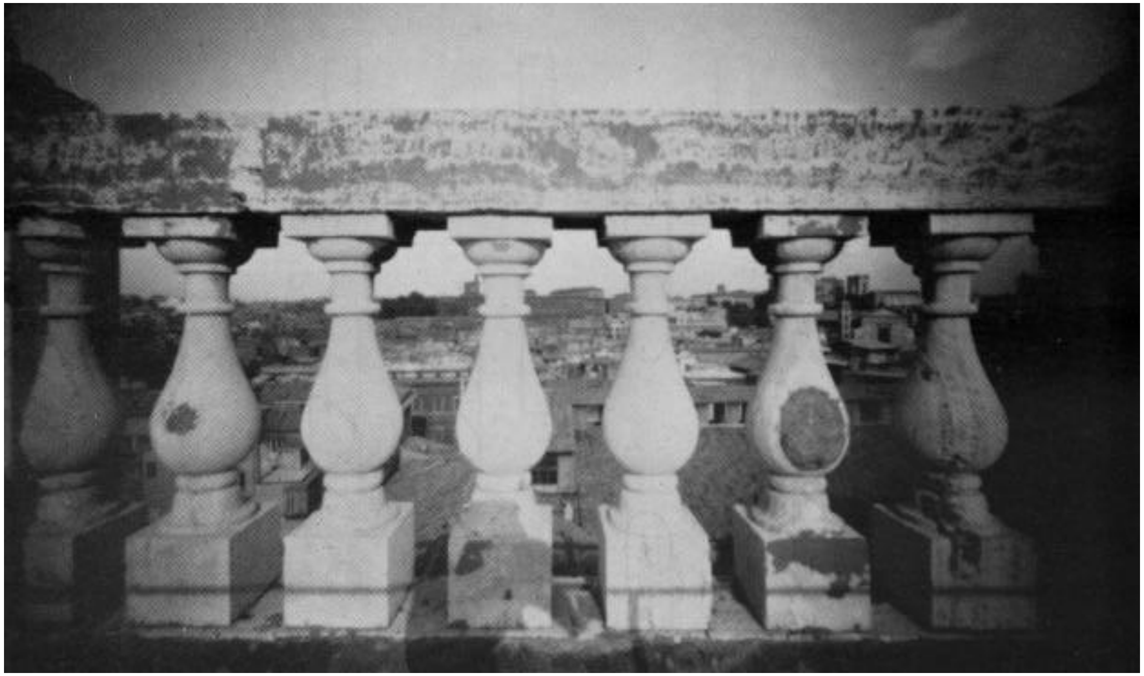


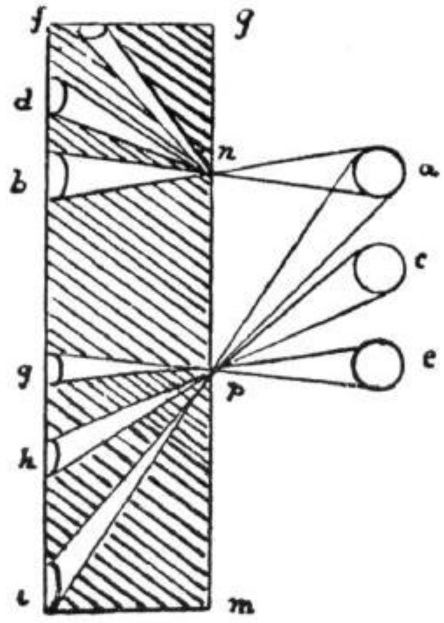
- Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

Properties of Projection

Objects on the periphery are expanded

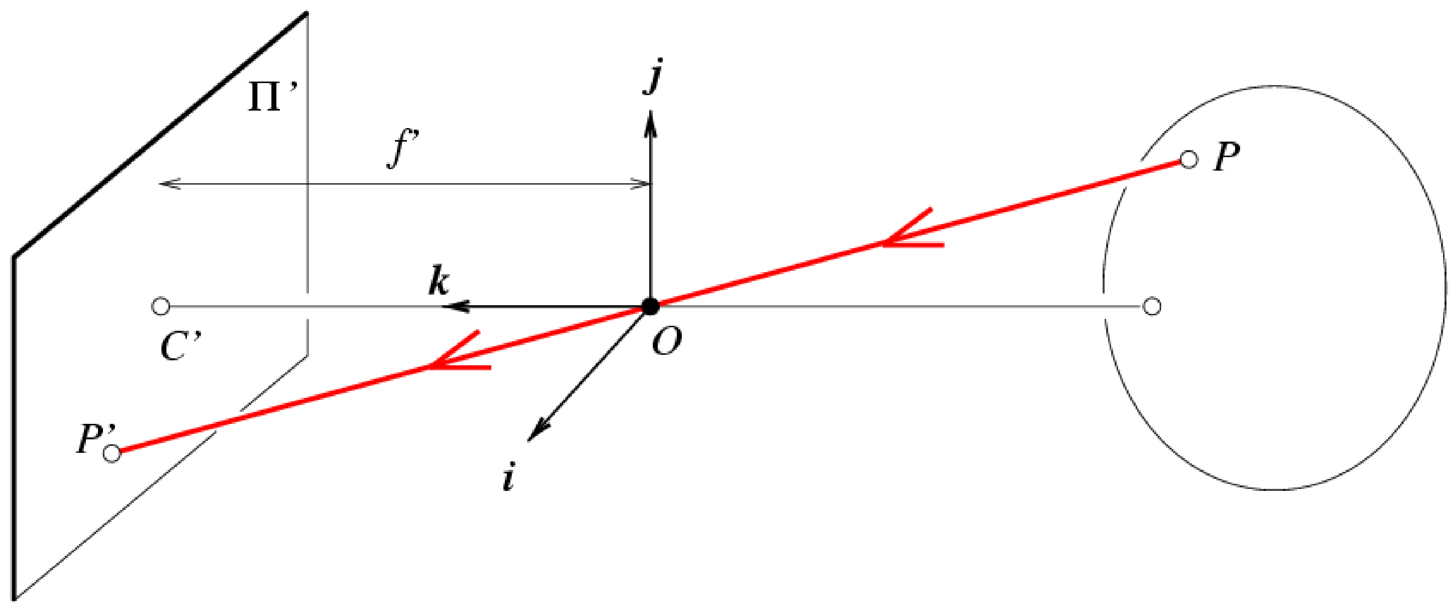
- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by DaVinci





Properties of Projection

- Degenerate cases
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image (with horizon).



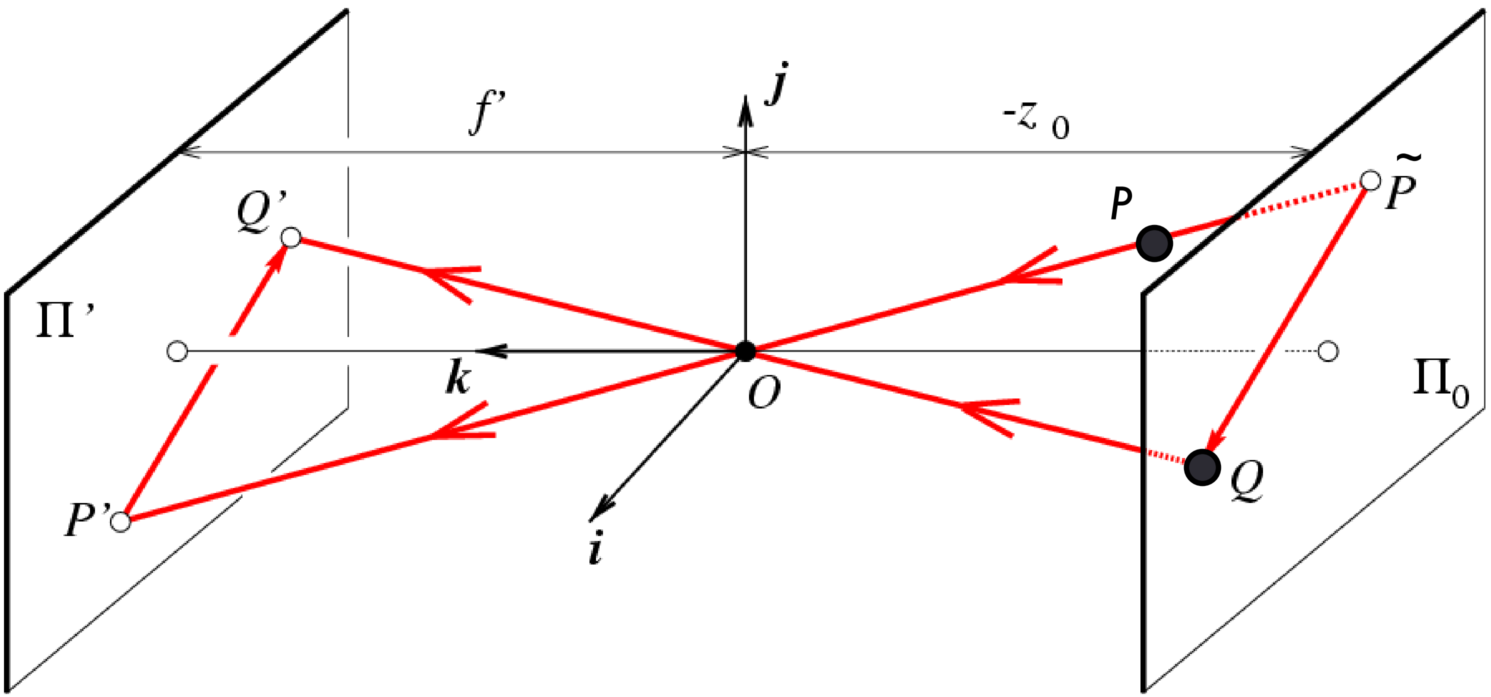
Outline

Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Weak perspective projection

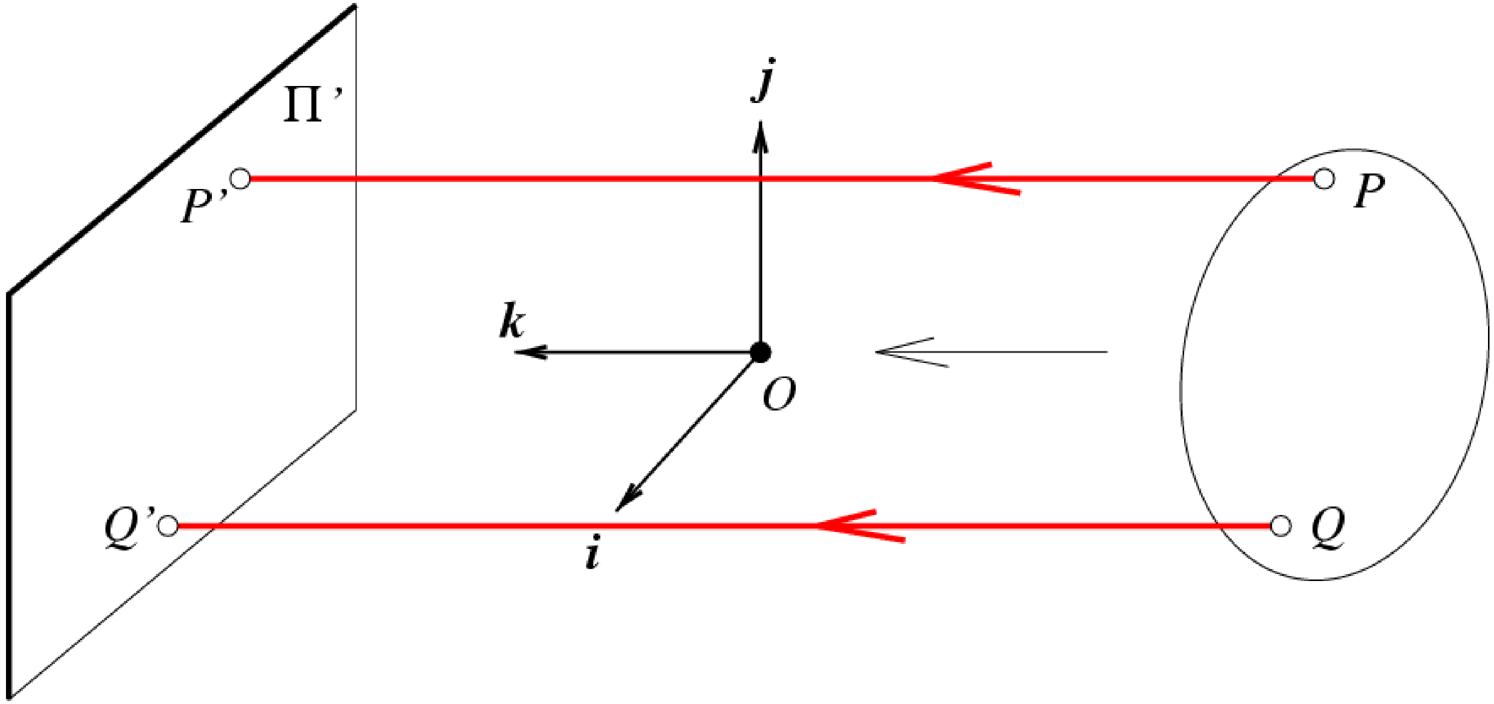
When the relative scene depth is small compared to its distance from the camera



$$\begin{cases} x' = -mx \\ y' = -my \end{cases}$$ where $m = -\dfrac{f'}{z_0}$ is the magnification.

Orthographic (affine) projection

When the camera is at a (roughly constant) distance from the scene



–Distance from center of projection to image plane is infinite

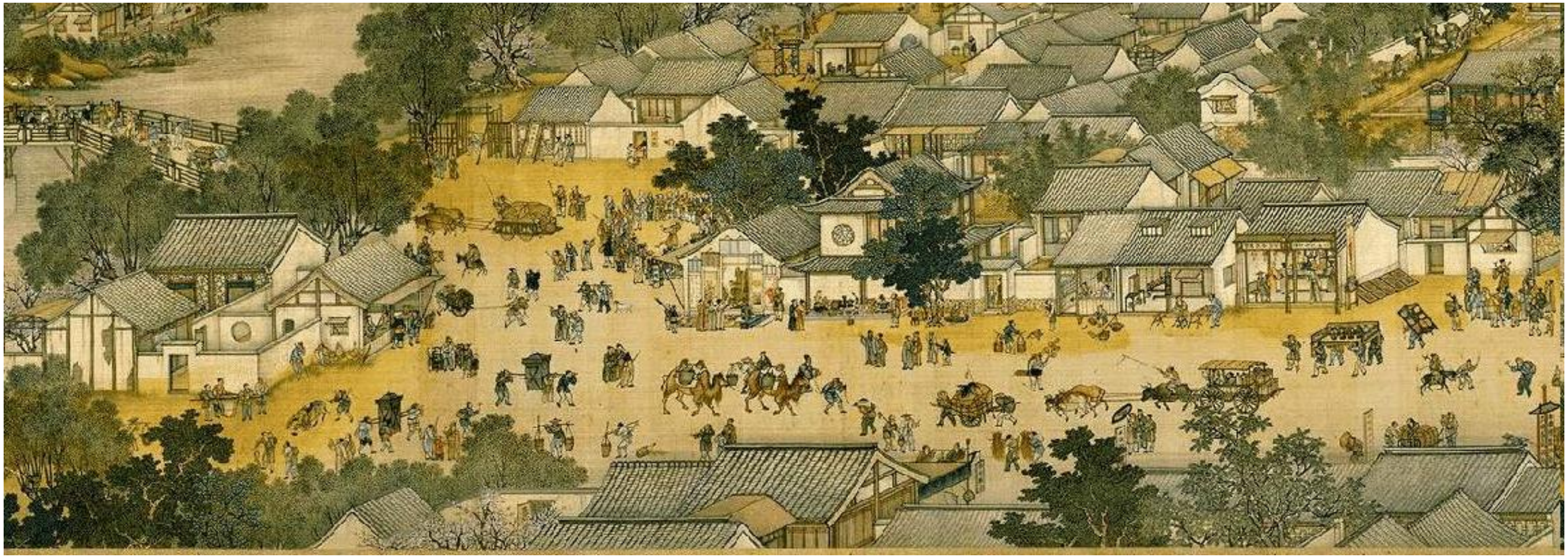
$$\begin{cases} x' = x \\ y' = y \end{cases}$$

Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.

- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.

Weak perspective projection



Qingming Festival by the Riverside Zhang Zeduan ~900 AD

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui
http://www.youtube.com/watch?v=mrFDGct4kH8

Next lecture

How to calibrate a camera?